Variables

declares param — using param

Function a → a + 1.

Function x → e

* a should be an int
* In OCaml, "a" is an ident

When a variable appears in an expr, it is being used

A use is bound to a declaration.

A variable is "bound" if it appears under the scope of a declaration.

A variable that is not bound is "free".

Function a → a = b

cow ⟹ cow

An expression that contains <u>any</u> free variables is "open".

No open expressions evaluate.

Let a = 1 In a + a ⟹ 2

Let a = 2 + 3 In a + a ⟹ 10

Replace x with v everywhere in e if it is free.

$e[v/x]$

$$\text{Let}\ \frac{e_1 \Rightarrow v_1 \qquad e_2[v_1/x] \Rightarrow v_2}{\text{Let } x = e_1 \text{ In } e_2 \Rightarrow v_2}$$

$4[5/a] = 4$

$a[5/a] = 5$

$b[5/a] = b$

$(\text{Let } a = 4 \text{ In } a)[5/a] = (\text{Let } a = 4 \text{ In } a)$

Let c = 4 In
Let c = c In
c

$(\text{Let } x = e_1 \text{ In } e_2)[v/x] = \text{Let } x = e_1[v/x] \text{ In } e_2$

$x \neq x'$

$(\text{Let } x = e_1 \text{ In } e_2)[v/x'] = \text{Let } x = e_1[v/x'] \text{ In } e_2[v/x']$

$(e_1 + e_2)[v/x'] = (e_1[v/x']) + (e_2[v/x'])$

Substituting a

Let a = 1 In a

Let a = a In 1

$(\text{Let } a = 1 \text{ In } a + b)[4/b]$

$= (\text{Let } a = 1 \text{ In } a + 4)$

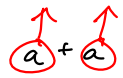
Function Application

e_1 e_2

↑ fn called ↑ arg passed

0. Eval e_1 to v_1
1. Eval e_2 to v_2
2. Call v_1
   a. Replace param in body with v_2
   b. Evaluate the result.

(Function a → a + a) 4



4 + 4

Write the application rule.

$e ::= \ldots \mid e_1\ e_2$

$$\frac{e_1 \Rightarrow v_1 \quad e_2 \Rightarrow v_2 \quad v_1 = \text{Function } x \to e_3 \quad e_3[v_2/x] \Rightarrow v_3}{e_1\ e_2 \Rightarrow v_3}$$

Let Rec

rec fn, name of param

$e ::= \ldots \mid$ Let Rec $x\ x = e$ In e ← rec fn

name of rec fn — name of param

Let Rec summate n =
If n = 0 Then 0
Else n + summate (n-1)
In
summate 3

$$\frac{e_2\left[\left(\text{Function } x_2 \to e_1\left[\text{Let Rec } x_1\ x_2 = e_1 \text{ In } x_1 / x_1\right]\right) / x_1\right] \Rightarrow v_3}{\text{Let Rec } x_1\ x_2 = e_1 \text{ In } e_2 \Rightarrow v_3}$$