CS46 practice problems 9

These practice problems are an opportunity for discussion and trying many different solutions. It is **not counted towards your grade**, and **you do not have to submit your solutions.** The purpose of this lab is to practice with Turing machines and our many accumulated techniques for proving things about Turing machines.

1. **Equal language checking for grammars.**

$$\mathrm{EQ}_{\mathrm{CFG}} = \{\langle G, H\rangle \mid G \text{ and } H \text{ are context-free grammars and } L(G) = L(H)\}$$

   (a) Show that $\mathrm{EQ}_{\mathrm{CFG}}$ is co-Turing-recognizable.[1]

   (b) Show that $\mathrm{EQ}_{\mathrm{CFG}}$ is undecidable.[2]
   (Note: this is why Automata Tutor for grammars ran thousands of test strings, instead of giving a definite answer!)

2. **Classifying languages.** For each of the following languages, is the language decidable?[3] Turing-recognizable? co-Turing-recognizable?

   Provided an argument for your answers. (Give the deciders/recognizers that you claim exist, and show why they work; if they do not exist, then prove why not.)

   You may consider the questions in any order, if proving one helps you with another.

   (a) $ALL_{\mathrm{TM}} = \{\langle M\rangle \mid M \text{ is a Turing machine and } L(M) = \Sigma^*\}$

   (b) $ODD_{\mathrm{TM}} = \{\langle M\rangle \mid M \text{ is a Turing machine and } L(M) \text{ contains no strings of even length}\}$

   (c) $E_{\mathrm{TM}} = \{\langle M\rangle \mid M \text{ is a Turing machine and } L(M) = \emptyset\}$

   (d) $HUNDRED_{\mathrm{TM}} = \{\langle M, w\rangle \mid M \text{ is a Turing machine and } M \text{ never moves its head past the } 100^{\text{th}} \text{ tape square during its computation on } w\}$

[1] Hint: Use nondeterminism.
[2] Hint: Theorem 5.13 shows $\mathrm{ALL}_{\mathrm{CFG}}$ is undecidable; you can use this result without proof.
[3] Hint: We already know that E_{TM} it is not decidable, Theorem 5.2.