CS46 practice problems 5

These practice problems are an opportunity for discussion and trying many different solutions. They are **not counted towards your grade**, and **you do not have to submit your solutions.** The purpose of these problems is to get more comfortable with CFGs, PDAs, and the pumping lemma for context-free languages. You can test your constructed grammars and PDAs on Automata Tutor, but keep in mind that they get pretty complicated, so you should discuss the sanity of your approach with your group first.

1. Use the pumping lemma for context-free languages to show that the following language over $\Sigma = \{a, \#\}$ is not context-free:

$$\{a^n \# a^{2n} \# a^{3n} \mid n \geq 0\}$$

2. Let $L_{\text{eq-len}} = \{x \# y \mid |x| = |y|\}$, where $x, y \in \{a, b\}^*$.

   (a) Design a grammar for $L_{\text{eq-len}}$.

   (b) Describe a PDA for $L_{\text{eq-len}}$.

3. Let $L_{\text{neq-len}} = \{x \# y \mid |x| \neq |y|\}$, where $x, y \in \{a, b\}^*$.

   (a) Design a grammar for $L_{\text{neq-len}}$.

   (b) Describe a PDA for $L_{\text{neq-len}}$.

4. Let $L_{\text{eq}} = \{x \# y \mid x = y\}$, where $x, y \in \{a, b\}^*$.

   Show that L_{eq} is not context-free using the pumping lemma for context-free languages.

5. Let $L_{\text{neq}} = \{x \# y \mid x \neq y\}$, where $x, y \in \{a, b\}^*$.

   (a) First, design a grammar or PDA for the language $\{a^p b^q b^p a^q \mid p, q \geq 0\}$. (Hint: it should be pretty simple.)

   (b) (Hint step. Next, figure out how to break up L_{neq} into subsets. Some subsets you should already have solved (above). For the remaining subset, find a way to write it which looks a lot like part (a).)

   (c) Design a grammar for L_{neq}.

   (d) Describe a PDA for L_{neq}.

   (Note: this is very complicated! You should use your earlier PDAs as "subroutines" to break this into more comprehensible parts.)

6. Use the pumping lemma for context-free languages to show that the following language is not context-free:

$$\{w \# t \mid w \text{ is a substring of } t, \text{ and } w, t \in \{a, b\}^*\}$$

Extra practice problems!

We now have a reasonable collection of interesting problems. For each of these languages that we've encountered recently, is it

(i) regular?

(ii) context-free, but not regular?

(iii) not even context-free?

Support your choice with a construction or proof. (You may have already done parts of this problem in the course of last week's lab/homework.) Remember that we have many labor-saving theorems and already-proven facts that you can cite. If you've already done a proof using a pumping lemma, try to do it again with closure properties.

7. $L_1 = \{w\overline{w} \mid \overline{w}$ is w with all as flipped to bs and all bs flipped to as$\}$ where $\Sigma = \{a, b\}$.
8. $L_2 = \{a^k u a^k \mid k \geq 1$ and $u \in \Sigma^*\}$ where $\Sigma = \{a, b\}$.
9. $L_3 = \{a^k b u a^k \mid k \geq 1$ and $u \in \Sigma^*\}$ where $\Sigma = \{a, b\}$.
10. $L_4 = \{w \mid w$ is unary for 10^n for some $n \geq 0\}$ where $\Sigma = \{1\}$.
11. $L_5 = \{w \mid w$ is decimal for 10^n for some $n \geq 0\}$ where $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.
12. $L_6 = \{a^n b^m a^m b^n \mid m, n \geq 0\}$ where $\Sigma = \{a, b\}$.
13. $L_7 = \{a^{m-n} \mid \frac{m}{n} = 5\}$ where $\Sigma = \{a, b\}$.
14. $L_8 = \{a^m b^n \mid m$ and n are prime factors of some integer $\leq 2022\}$ where $\Sigma = \{a, b\}$.
15. $L_9 = \{w \mid w$ is not a palindrome$\}$ where $\Sigma = \{a, b\}$.
16. $L_{10} = \{w \mid w = x_1 \# x_2 \# \cdots \# x_k$ for $k \geq 0$, each $x_i \in L(a^*)$, and $x_i \neq x_j$ for $i \neq j\}$, where $\Sigma = \{a, \#\}$.