CS46 practice problems 4

These practice problems are an opportunity for discussion and trying many different solutions. They are **not counted towards your grade**, and **you do not have to submit your solutions.** The purpose of these problems is to get more comfortable with the pumping lemma for regular languages, as well as using and thinking about context-free grammars. You can try out your context-free grammars on Automata Tutor.

1. For each of the following languages, is the language regular? Support your claim with a proof.

   (a) Define $f(w) =$ flip all bs to as and flip all as to bs in $w\}$ for $f : \{a, b\}^* \to \{a, b\}^*$. Consider $L_1 = \{f(w) \mid w \in L\}$ where L is some regular language. (This question is the same as asking: are regular languages closed under f?)

   (b) $L_2 = \{w\overline{w} \mid \overline{w}$ is w with all as flipped to bs and all bs flipped to as$\}$ where $\Sigma = \{a, b\}$.

   (c) $L_3 = \{w \mid w$ is unary for 10^n for some $n \geq 0\}$ where $\Sigma = \{1\}$.

   (d) $L_4 = \{w \mid w$ is decimal for 10^n for some $n \geq 0\}$ where $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

   (e) $L_5 = \{a^m b^n \mid m$ and n are prime factors of some integer $\leq 2022\}$ where $\Sigma = \{a, b\}$.

2. Consider the grammar G with rules: $\begin{cases} S & \to aSa \mid bT \\ T & \to aT \mid bT \mid \varepsilon \end{cases}$

   Figure out two words $\in L(G)$ and two words $\notin L(G)$. (You can check your work on Automata Tutor.) What is the language $L(G)$?

3. Many programming languages use braces {} , brackets [ ], and parentheses () to group functions, blocks, classes, etc. These braces, brackets, and parentheses must be balanced in the sense that you cannot have a closing brace without a previous matching opening brace, all open braces must eventually have a matching closing brace, and you cannot close a brace with an unmatched open brace "inside."

   The following examples are legal: ()(), (({})[{}]{{}}), and {}[()].

   The following examples are not legal: [(]), ((, and (}.

   Design a context free grammar that generates balanced statements containing braces, brackets, and parentheses. (You can check your work on Automata Tutor.)

   Outline a formal argument proving that your grammar is correct.

4. Give a context-free grammar over $\Sigma = \{a, b\}$ generating

   $$L = \{w \in \Sigma^* \mid w \text{ contains more } a\text{s than } b\text{s}\}$$

   (You can check your work on Automata Tutor.)

5. Give a context-free grammar generating:

   $$L = \{wcx \mid w^R \text{ is a substring of } x, \text{ where } w, x \in \{a, b\}^*\} \subseteq \{a, b, c\}^*$$

   (You can check your work on Automata Tutor.)

6. Let $L_{\text{happy}} = \{w \mid w$ contains twice as many ☺ s as ☹ s$\}$ be a language over $\Sigma = \{$☺, ☹$\}$.

   (a) Prove that L_{happy} is not regular.

   (b) Prove that L_{happy} is context-free. (Construct a grammar generating L_{happy} and check it on Automata Tutor, or, if you are feeling adventurous and confident, construct a pushdown automata recognizing L_{happy}.)

7. If you've finished all the above problems, then consider:

   - For each of the languages in problem 1 that you said were *not* regular: is that language context-free? Support your answer with an outline of an argument or construction.
   - Give an informal English description of a PDA for the languages where you built a grammar.