CS46 practice problems 2

These practice problems are an opportunity for discussion and trying many different solutions. They are **not counted towards your grade**, and **you do not have to submit your solutions.** The purpose of these problems is to get more comfortable with DFAs and with using the Automata Tutor site. You may want to try to solve these problems on paper *first*, then trying with the online tool. Once you are ready to test your solutions, the site will give you feedback on how to improve your solution. For each practice problem, you are allowed unlimited attempts.

For all of these problems, $\Sigma = \{a, b\}$.

0. Go to Automata Tutor and create a login (use your preferred first and last name, as we will use this tool for actual graded problems as well). Enroll in this course with:
   Course ID: CS46-2022S
   Course Password: HMILPTJV

1. Construct a DFA for the language $\{w \mid w \text{ contains the substring } ab\}$.

2. Construct a DFA for the language $\{w \mid w \text{ does } \textit{not} \text{ contain the substring } ab\}$.

3. Construct a DFA for the language $\{w \mid w \text{ contains the substring } baba\}$.

4. Construct a DFA for the language $\{w \mid w \text{ does } \textit{not} \text{ contain the substring } baba\}$.

5. Construct a DFA for the language $\{aa, abba\}$.

   You might consider breaking this problem into pieces:

   (a) Construct a DFA for the language $\{aa\}$.
   (b) Construct a DFA for the language $\{abba\}$.
   (c) Use the proof idea from theorem 1.25 (regular languages are closed under union) to construct a new DFA for the union language from your two simpler DFAs.

6. Construct a DFA for the language $\{w \mid w \text{ contains exactly two } a\text{s and at least two } b\text{s}\}$.

   You might consider breaking this problem into pieces:

   (a) Construct a DFA for the language $L_1 = \{w \mid w \text{ contains exactly two } a\text{s}\}$
   (b) Construct a DFA for the language $L_2 = \{w \mid w \text{ contains at least two } b\text{s}\}$.
   (c) We want to construct a DFA for $L_1 \cap L_2$, so we can use an idea like the footnote (page 46) on the proof of theorem 1.25 to construct the states and transitions for this new DFA.

7. Construct a DFA for the language $L = \{\varepsilon\}$ over the alphabet $\Sigma = \{a, b\}$.

8. Construct a DFA for the language $L = \{w \mid w \text{ does not contain exactly two } a\text{s}\}$ over the alphabet $\Sigma = \{a, b\}$.

9. Construct a DFA for the language $L = \{w \mid 3 \leq |w| \leq 5\}$ over the alphabet $\Sigma = \{a, b\}$.

10. Construct a DFA for the language $L = \{w \mid a \text{ appears } k \text{ times in } w \text{ where } k + 1 \text{ is divisible by } 3\}$ over the alphabet $\Sigma = \{a, b\}$.

11. Construct a DFA for the language $L = \{w \mid \text{ every } b \text{ in } w \text{ is immediately followed by two } a\text{s}\}$ over the alphabet $\Sigma = \{a, b\}$.

12. **Serious challenge.** With only 3 attempts: Construct a DFA for the language

$$L = \{w \mid w \text{ is a binary number equal to 1 mod 3}\}$$

over alphabet $\Sigma = \{0, 1\}$. (So $0 \notin L$, $1 \in L$, $100 \in L$, etc.)

(**Advice:** You absolutely need to figure this one out on paper first, or you will run out of attempts.)

13. (discuss this with one or two other students)

Let $\Sigma = \{a, b, c, \ldots, z\}$. For any language $A \subseteq \Sigma^*$, let the **contrary** of A be defined as:

$$\text{contrary}(A) = \{antiw \mid w \in A\}$$

For example, if $A = \{unicorn, pony, tricycle\}$, then

$$\text{contrary}(A) = \{antiunicorn, antipony, antitricycle\}$$

Prove that the class of regular languages is closed under the "contrary" operator. (That is, prove that if A is regular, then contrary(A) is regular. You should describe how to construct a machine that recognizes contrary(A), define all elements of your machine $M = (Q, \Sigma, \delta, q_0, F)$, and argue why this machine recognizes contrary(A).)

14. (discuss this with one or two other students)

Consider the language $C = \text{op}(A, B)$ where "op" is some operation that regular languages are closed under. Suppose we know the following about A and C. What, if anything, can we conclude about B?

(You should support your answer with a brief explanation.)

(a) A is regular and C is regular.
(b) A is regular and C is not regular.
(c) A is not regular and C is regular.
(d) A is not regular and C is not regular.