CS46 practice problems 10

These practice problems are an opportunity for discussion and trying many different solutions. It is **not counted towards your grade**, and **you do not have to submit your solutions.** The purpose of these problems is to get more comfortable with reasoning and writing proofs about decidability, recognizability, and co-recognizability.

   If you are stumped or looking for guidance, **ask**.

1. Consider the language $L = \{\langle M, w \rangle \mid M$ is a single-tape TM that never modifies the portion of the tape that contains the original input $w\}$.

   (a) Show that L is co-Turing-recognizable, by briefly describing the elements of $\overline{L}$ and then describing a recognizer for $\overline{L}$.

   (b) Is L decidable? Prove your answer.
      Note that if you can show that L is Turing-recognizable, then you can apply Theorem 4.22 and part (a) to show L is decidable.

2. For each of the following languages, review if the language is decidable, Turing-recognizable, co-Turing-recognizable, or none of these. A_{DFA}, A_{CFG}, A_{TM}, E_{DFA}, E_{CFG}, E_{TM}, ALL_{DFA}, ALL_{CFG}, ALL_{TM}, EQ_{DFA}, EQ_{CFG}, EQ_{TM}.

3. Consider the language of Turing machines which only accept strings consisting of as and bs:

   $$\mathrm{TargetGPA}_{\mathrm{TM}} = \{\langle M \rangle \mid M \text{ is a Turing machine and } L(M) \subseteq L((a \cup b)^*)\}$$

   Is this language regular? context-free? decidable? recognizable? co-recognizable?

   You may consider these parts in any order. (Some orders will be more helpful than others.) Support your answer for each part with a proof!

4. Consider the language of deciders:

   $$\mathrm{DECIDER}_{\mathrm{TM}} = \{\langle M \rangle \mid M \text{ is a decider}\}$$

   (a) Recall that $L_{\mathrm{TM}} = \{\langle M \rangle \mid M$ is a Turing machine$\}$, so $\mathrm{DECIDER}_{\mathrm{TM}} \subseteq L_{\mathrm{TM}}$.
      Does Rice's Theorem apply to $\mathrm{DECIDER}_{\mathrm{TM}}$?

   (b) Show that $\mathrm{DECIDER}_{\mathrm{TM}}$ is undecidable.

   (c) Show that $\mathrm{DECIDER}_{\mathrm{TM}}$ is both not recognizable and not co-recognizable.

ONLY IF you finish problems 1-4, look at problem 5. The busy beaver problem is interesting, and famous, and challenging. It is also *much, much* more complicated than any problem I would *ever* ask on a homework or exam. It's here for your intellectual enjoyment!