CS46 lab 8

This homework is due at 11:59pm on Sunday, March 20. This is a 10-point homework.

For this homework, you will work with a partner. It's ok to discuss approaches at a high level with other students, your discussions should be just with your partner. The only exception to this rule is work you've done with another student *while in lab*. In this case, note who you've worked with and what parts were solved during lab. Your partnership's write-up and code is your own: do not share it, and do not read other teams' write-ups. If you use any out-of-class references (anything except class notes, the textbook, or asking Lila), then you **must** cite these in your post-homework survey. Please refer to the course webpage or directly ask any questions you have about this policy.

The main **learning goal** of this homework is to work with and think about Turing machines and decidability. You should feel free as always to cite and use techniques and theorems from class or the textbook.

1. Prove that the following language is decidable:

$$\{\langle M \rangle \mid M \text{ is a DFA and } \forall w, \text{ if } w \in L(M) \text{ then } w^R \in L(M)\}$$

2. Show that ALL_{DFA} is decidable, where ALL_{DFA} is defined as:

$$ALL_{DFA} = \{\langle A \rangle \mid A \text{ is a DFA and } L(A) = \Sigma^*\}$$

   (Hint: you may want to refer to the textbook for some other decidable languages related to DFAs, and recall back when we proved things like "regular languages are closed under intersection". I suggest you also look at the textbook solution for showing that $INFINITE_{DFA}$ is decidable (Sipser 4.10), which is different from what any group came up with during lab.)

3. Show that every infinite Turing-recognizable language has an infinite decidable subset.

4. **Computable functions.** Recall that a function $f : \Sigma^* \to \Sigma^*$ is **computable** if some Turing machine M, on every w, halts and accepts with just $f(w)$ on its tape.

   (a) Let $f : \Sigma^* \to \Sigma^*$ be a partial computable function which is one-to-one and onto. Prove that f^{-1} is a total computable function.

   (b) Show that if functions f and g are computable, then their composition $f \circ g$ is computable.

5. **(extra challenge)** (Sipser 3.17) Let $B = \{\langle M_1 \rangle, \langle M_2 \rangle, \ldots\}$ be a Turing-recognizable language consisting of Turing machine descriptions. Show that there is a decidable language C consisting of Turing machine descriptions such that every machine described in B has an equivalent machine in C, and every machine described in C has an equivalent machine in B.

6. **(extra challenge)** A_{TM} is a language consisting of descriptions of Turing machines, and it is Turing-recognizable. Why does the previous question not imply that A_{TM} is decidable?