CS46 lab 6

This homework is due at 11:59PM on Sunday, February 27. This is an **8 point** homework.

For this homework, you will work with a partner. It's ok to discuss approaches at a high level with other students, your discussions should be just with your partner. The only exception to this rule is work you've done with another student *while in lab*. In this case, note who you've worked with and what parts were solved during lab. Your partnership's write-up and code is your own: do not share it, and do not read other teams' write-ups. If you use any out-of-class references (anything except class notes, the textbook, or asking Lila), then you **must** cite these in your post-homework survey. Please refer to the course webpage or directly ask any questions you have about this policy.

The main **learning goal** of this homework is to work with and think about context-free languages, and to practice using the Pumping Lemma for Context-Free Languages.

1. The textbook (example 2.38) shows that the language

$$L = \{ww \mid w \in \{0,1\}^*\}$$

   is not context-free. Prove that $\overline{L}$ is context-free. (Note: This shows that the context-free languages are *not* closed under complementation! We also saw a counterexample of this in practice problems 5. Also note: this language's alphabet does not include #, so it is different from the practice problem you discussed. Be careful! We've seen examples where one character makes a big difference.)

2. Using the stack in a PDA can be subtle. One way to see this is examine two languages that are *very* similar, and show that they require different computational power to recognize. (Hint: the difference is going to be in the usage of the stack!)

   Note that:

   - i and j are not necessarily distinct in part (a)
   - any palindrome x satisfies $x = x^R$
   - $|x_i|$ can be zero for any i

   (a) Give a context-free grammar that generates (or pushdown automata that recognizes) the language:

$$\{t_1\#t_2\#\cdots\#t_k \mid k \geq 1,\ \text{each } t_i \in \{a,b\}^*,\ \text{and } t_i = t_j^R \text{ for some } i, j\}$$

   You do not have to prove the correctness of your grammar/PDA, but you should give a high-level explanation of why/how you designed it, and why both directions of proof should work.

   (b) Use the pumping lemma to show that the following language is not context-free:

$$\{t_1\#t_2\#\cdots\#t_k \mid k \geq 2,\ \text{each } t_i \in \{a,b\}^*,\ \text{and } t_i = t_j \text{ for some } i \neq j\}$$

   (c) **(extra challenge)** Use closure properties to show that the language from part (b) is not context-free.

3. **(extra challenge)** Inspired by genetics, define the CROSSOVER operator as follows:

$$\text{CROSSOVER}(A, B) = \{x_1y_2, x_2y_1 \mid x_1x_2 \in A, y_1y_2 \in B, \text{ and } |x_1| = |x_2| = |y_1| = |y_2|\}$$

   So for every pair of equal-length strings $x_1x_2 \in A$ and $y_1y_2 \in B$, we add two strings to the crossover language by cutting them in half and recombining them.

   For example, if $A = \{a, aa, aabb\}$ and $B = \{\varepsilon, cc, caca, aacaa\}$ then $\text{CROSSOVER}(A, B) = \{ac, ca, aaca, cabb\}$.

   Show that if A and B are regular, then $\text{CROSSOVER}(A, B)$ is not necessarily regular.

   Show that if A and B are regular, then $\text{CROSSOVER}(A, B)$ is context-free.