CS46 Homework 3

This homework is due at 11:59PM on Sunday, February 6. Write your solution using LaTeX. Submit this homework using **github**. This is a **10 point** homework.

This is an individual homework. It's ok to discuss approaches at a high level. In fact, I encourage you to discuss general strategies. However, you should not reveal specific details of a solution, nor should you show your written solution to anyone else. Your write-up is your own. If you use any out-of-class references (anything except class notes, the textbook, or asking Lila), then you **must** cite these in your post-homework survey. Please refer to the course webpage or ask me any questions you have about this policy.

The main **learning goal** of this homework is to develop the skills to design, understand, and analyze DFAs, and to think about the class of regular languages in general.

Part 1 — These problems should be completed[1] on Automata Tutor. You are allowed **three attempts** at each problem. I recommend that you *first* try to solve the problems on paper, *then* use the site to debug your solutions.

1. Construct a DFA for the language $\emptyset$ over alphabet $\Sigma = \{0, 1\}$.
2. Construct a DFA for the language $\{\varepsilon, 0\}$ over alphabet $\Sigma = \{0, 1\}$.
3. Construct a DFA for the language $\{w \mid w \text{ is either } a \text{ or } b\}$ over alphabet $\Sigma = \{a, b\}$.
4. Construct a DFA for the language $\{w \mid w \text{ is any string except } a \text{ or } b\}$ over alphabet $\Sigma = \{a, b\}$.
5. Construct a DFA for the language $\{w \mid w \text{ contains at least three 1s}\}$ over alphabet $\Sigma = \{0, 1\}$.
6. Construct a DFA for the language $\{w \mid \text{every } a \text{ in } w \text{ is immediately followed by a } b\}$ over alphabet $\Sigma = \{a, b\}$.
7. Construct a DFA for the language $\{w \mid b \text{ occurs } n \text{ times in } w, \text{ where } n \text{ is divisible by } 3\}$ over alphabet $\Sigma = \{a, b\}$.
8. Construct a DFA for the language $\{w \mid \text{ length of } w \leq 5\}$ over alphabet $\Sigma = \{a, b\}$.
9. Construct a DFA for the language $\{w \mid w \text{ contains at least two 0s and at most one 1}\}$ over alphabet $\Sigma = \{0, 1\}$.
10. Construct a DFA for the language $L = \{w \mid \text{ every odd position of } w = w_1 w_2 w_3 \ldots w_n \text{ is a } 1\}$ over the alphabet $\Sigma = \{0, 1\}$.
11. Construct a DFA for the language $L = \{w \mid w \text{ is any non-empty string}\}$ over the alphabet $\Sigma = \{0, 1\}$.
12. Construct a DFA for the language $L = \{w \mid w \text{ begins and ends with the same symbol}\}$ over the alphabet $\Sigma = \{0, 1\}$. This language includes the empty string.

[1] If you want to use late days on this assignment, you will need to submit solutions to these problems via github. The automatatutor site has only one deadline.

13. **Extra credit.** Construct a DFA for the language $L = \{w \mid w \text{ is a binary number equal to 2 mod 5}\}$ over alphabet $\Sigma = \{0, 1\}$. (So $0 \notin L$, $10 \in L$, $100 \notin L$, etc.)

Part 2 — These problems should be typeset in LaTeX and submitted using **github**.

14. Let $\Sigma = \{a, b, c, \ldots, z\}$. For any language $A \subseteq \Sigma^*$, let the **contrary** of A be defined as:

$$\text{contrary}(A) = \{antiw \mid w \in A\}$$

For example, if $A = \{unicorn, pony, tricycle\}$, then

$$\text{contrary}(A) = \{antiunicorn, antipony, antitricycle\}$$

Prove that the class of regular languages is closed under the "contrary" operator. (That is, prove that if A is regular, then contrary(A) is regular. You should describe how to construct a machine that recognizes contrary(A), define all elements of your machine $M = (Q, \Sigma, \delta, q_0, F)$, and argue why this machine recognizes contrary(A).)

15. Consider the language $C = \text{op}(A, B)$ where "op" is some operation that regular languages are closed under. Suppose we know the following about A and C. What, if anything, can we conclude about B?

(You should support your answer with a brief explanation. Even though we have not yet seen any specific languages that are *not* regular, you can approach this problem using just the definition of "regular language" and "closed".)

(a) A is regular and C is regular.
(b) A is regular and C is not regular.
(c) A is not regular and C is regular.
(d) A is not regular and C is not regular.

16. (**extra credit**) For languages A and B, let the **perfect shuffle** of A and B be the language:

$$\{w \mid w = x_1 y_1 x_2 y_2 \cdots x_k y_k \text{ where } x_1 \cdots x_k \in A \text{ and } y_1 \cdots y_k \in B \text{ and each } x_i, y_i \in \Sigma\}$$

Prove that the class of regular languages is closed under perfect shuffle.