CS46 lab 2

This homework is due at 11:59PM on Sunday, January 30. Write your solution using LaTeX. Submit this homework using **github**. This is a **10 point** homework.

This is an individual homework. It's ok to discuss approaches at a high level. In fact, I encourage you to discuss general strategies. However, you should not reveal specific details of a solution, nor should you show your written solution to anyone else. Your write-up is your own. If you use any out-of-class references (anything except class notes, the textbook, or asking Lila), then you **must** cite these in your post-homework survey. Please refer to the course webpage or ask me any questions you have about this policy.

The main **learning goal** of this homework is to develop the skills to work with the notation and terminology about sets, and start thinking about DFAs.

1. For each of the following statements, indicate whether it is true or false:

   (a) $\emptyset \in \emptyset$

   (b) $\emptyset \subseteq \emptyset$

   (c) $\emptyset \in \{\emptyset\}$

   (d) $\emptyset \subseteq \{\emptyset\}$

   (e) $\{\{\emptyset\}\} \subseteq \{\emptyset, \{\emptyset\}\}$

   (f) $\{\{\emptyset\}\} \subseteq \{\{\emptyset, \{\emptyset\}\}\}$

2. Let Σ be an alphabet (a set of letters). We define Σ^* as the set of all strings using letters from Σ. Let $\mathcal{C}$ be a collection of sets which are all subsets of Σ^*. We are given that $\Sigma^* \in \mathcal{C}$.

   Assume that $\mathcal{C}$ is closed under the operation set difference. (So if $A \in \mathcal{C}$ and $B \in \mathcal{C}$, then $A \backslash B \in \mathcal{C}$.)

   Using direct proof, show that:

   (a) If $A \in \mathcal{C}$, then $\bar{A} \in \mathcal{C}$. ($\mathcal{C}$ is closed under complement.)

   (b) If $A \in \mathcal{C}$ and $B \in \mathcal{C}$, then $A \cap B \in \mathcal{C}$. ($\mathcal{C}$ is closed under intersection.)

   (c) If $A \in \mathcal{C}$ and $B \in \mathcal{C}$, then $A \cup B \in \mathcal{C}$. ($\mathcal{C}$ is closed under union.)

   Your proofs should be fully formal, with all steps of explanation written out.

3. Write a concise English description of the language recognized by DFA M_1.

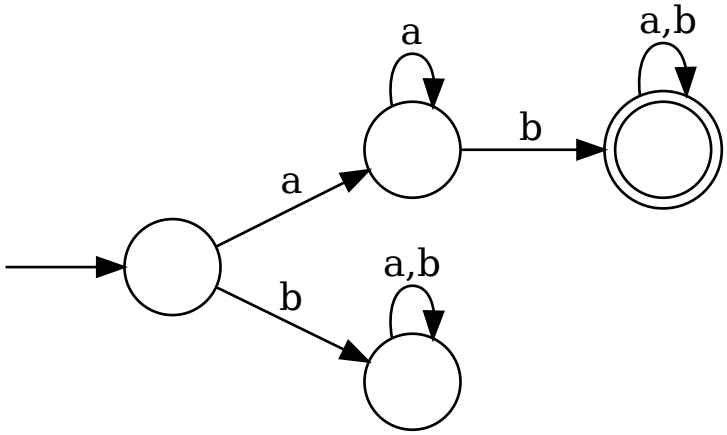

Figure 1: DFA M_1

4. Write a concise English description of the language recognized by DFA M_2.

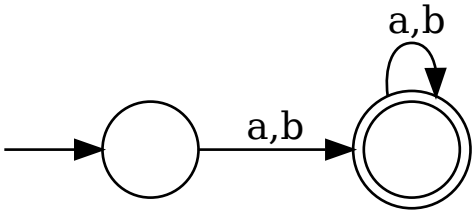

Figure 2: DFA M_2

5. Write a concise English description of the language recognized by DFA M_3.

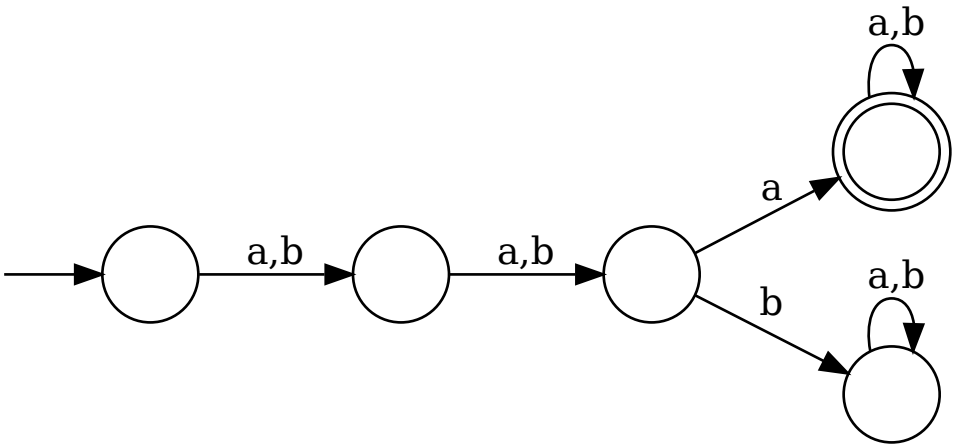

Figure 3: DFA M_3

6. **(extra credit)** Formally prove that $n^2 + n$ is divisible by 2 for all $n \in \mathbb{N}$ using induction.
7. **(extra credit)** Formally prove that $n^2 + n$ is divisible by 2 for all $n \in \mathbb{N}$ using direct proof.