CS46 lab 11

This homework is due at 11:59pm on Sunday, April 10. This is a 10-point homework.

For this homework, you will work with a partner. It's ok to discuss approaches at a high level with other students, but your detailed discussions should be only with your partner. The only exception to this rule is work you've done with another student *while in lab*. In this case, note who you've worked with and what parts were solved during lab. Your partnership's write-up and code is your own: do not share it, and do not read other teams' write-ups. If you use any out-of-class references (anything except class notes/the textbook/asking Lila), then you **must** cite these in your post-homework survey. Please refer to the course webpage or directly ask any questions you have about this policy.

The main **learning goal** of this homework is to work with and think about Turing machines and decidability, and to start adding runtime considerations to the analysis of languages that we've been developing all semester.

1. Consider the language of deciders:

   $$\textsc{Decider}_{\text{TM}} = \{\langle M \rangle \mid M \text{ is a decider}\}$$

   Show that $\textsc{Decider}_{\text{TM}}$ is undecidable, unrecognizable, and not co-recognizable.

2. Use the definition of big-O to prove that:

   (a) $2^n = O(5^n)$.

   (b) $n^2 \log n = O(n^3)$.

   (c) $\frac{1}{7} \cdot 3^n \neq O(n^2)$. (This should be a proof by contradiction.)

3. (Sipser 7.9) A **triangle** in a graph is three nodes that are all connected to each other by edges. Show that $\textsc{Triangle} \in \text{P}$, where

   $$\textsc{Triangle} = \{\langle G \rangle \mid G \text{ is a graph that contains a triangle}\}$$

   Refer back to chapter 0 for the details of the definition of a graph. You should be specific in your construction, as you will need to analyze the runtime of each line of your Turing machine.