

CS46 lab 10

This homework is due at 11:59pm on Sunday, April 3. This is a 10-point homework.

For this homework, you will work with a partner. It's ok to discuss approaches at a high level with other students, but your detailed discussions should be only with your partner. The only exception to this rule is work you've done with another student *while in lab*. In this case, note who you've worked with and what parts were solved during lab. Your partnership's write-up and code is your own: do not share it, and do not read other teams' write-ups. If you use any out-of-class references (anything except class notes/the textbook/asking Lila), then you **must** cite these in your post-homework survey. Please refer to the course webpage or directly ask any questions you have about this policy.

The main **learning goal** of this homework is to work with and think about Turing machines and decidability. You should feel free as always to cite and use techniques and theorems from class or the textbook. The focus here is to **write complete, thorough, convincing proofs**.

Provided an argument for your answers. (Give the deciders/recognizers that you claim exist, and show why they work; if they do not exist, then prove why not.)

You may consider the questions in any order, if proving one helps you with another.

1. $ALL_{TM} = \{\langle M \rangle \mid M \text{ is a Turing machine and } L(M) = \Sigma^*\}$
 - (a) Is ALL_{TM} decidable?
 - (b) Is ALL_{TM} recognizable?
 - (c) Is ALL_{TM} co-recognizable?

2. $ODD_{TM} = \{\langle M \rangle \mid M \text{ is a Turing machine and } L(M) \text{ contains no strings of even length}\}$
 - (a) Is ODD_{TM} decidable?
 - (b) Is ODD_{TM} recognizable?
 - (c) Is ODD_{TM} co-recognizable?

3. $HUNDRED_{TM} = \{\langle M, w \rangle \mid M \text{ is a Turing machine and } M \text{ never moves its head past the } 100^{\text{th}} \text{ tape square during its computation on } w\}$
 - (a) Is $HUNDRED_{TM}$ decidable?
 - (b) Is $HUNDRED_{TM}$ recognizable?
 - (c) Is $HUNDRED_{TM}$ co-recognizable?

For an example of a good level of thoroughness in construction descriptions and proof arguments, there is a write-up of this analysis (decidability/recognizability/co-recognizability) for E_{TM} on the course website, using every method I could think of. Please read it and feel free to use it as a template when writing your own proofs.