Positional Notation: Decimal Base 10

A number, written as the sequence of digits

$d_n d_{n-1} \ldots d_2 d_1 d_0$

where d is in {0,1,2,3,4,5,6,7,8,9},

represents the value:

$$[d_n * 10^n] + [d_{n-1} * 10^{n-1}] + \ldots + [d_2 * 10^2] + [d_1 * 10^1] + [d_0 * 10^0]$$

64025 =

$6 * 10^4 + 4 * 10^3 + 0 * 10^2 + 2 * 10^1 + 5 * 10^0$

60000 + 4000 + 0 + 20 + 5

Binary: Base 2

Used by computers: Indicated by prefixing number with **0b**

A number, written as the sequence of digits in {0,1}

$$[d_n * 2^n] + [d_{n-1} * 2^{n-1}] + \ldots + [d_2 * 2^2] + [d_1 * 2^1] + [d_0 * 2^0]$$

- 10101: $1*2^4 + 0*2^3 + 1*2^2 + 0*2^1 + 1*2^0$

  = 16 + 0 + 4 + 0 + 1 = 21

Different Representations

- Binary: base 2 digits [0,1]
- Decimal: base 10 digits [0, 1, …, 9]
- Hexadecimal: base 16 digits [0, …,9,a,b,c,d,e,f]

Relationship between Binary and Hexadecimal: 16 is 2^4

- each hex digit is unique permutation of 4 binary digits

  0000: 0 0001:1 0010:2 0011:3 0100:4 0101:5 0110:6 0111:7

  1000: 8 1001:9 1010:a 1011:b 1100:c 1101:d 1110:e 1111:f

Why hex? Shorthand for binary that is easier for humans to read

0011111011111010 -> 0011 1110 1111 1010 -> 0x 3 e f a

Let's try some more examples: Convert the following binary number to decimal and hexadecimal

1. 10100101

2. 00001111

What is the value of 357 in binary?

8 7654 3210 → digit position

A. 1 0110 0011
B. 1 0110 0101
C. 1 0110 1001
D. 1 0111 0101
E. 1 1010 0101

$2^0 = 1$, $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$,

$2^5 = 32$, $2^6 = 64$, $2^7 = 128$, $2^8 = 256$

Let's try some more examples (note down if you get a carry out)

```
  0100      4        1111      15
+ 0100    + 4      + 0001    +  1
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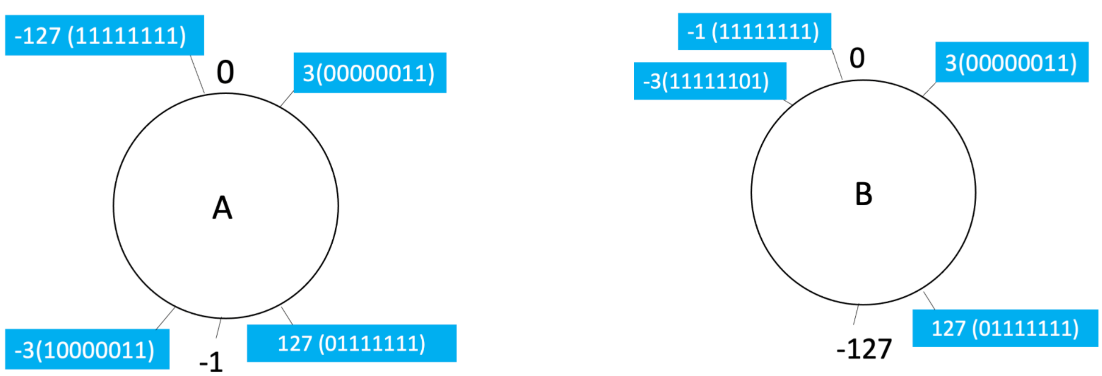
in binary 1+1 = 10
"0 carry the 1" out

<u>Carry out is indicative of something having gone wrong when adding unsigned values</u>

Suppose we want to support signed values (positive and negative) in 8 bits, where should we put -1 and -127 on the circle? Why?



C: Put them somewhere else.

If we interpret 11001 as a two's complement number, what is the value in decimal?

Each two's compliment number is now:

$$[-2^{n-1}*d_{n-1}] + [2^{n-2}*d_{n-2}] +...+ [2^1*d_1] + [2^0*d_0]$$

[No Title]

A. -2

B. -7

C. -9

D. -25

Let's try some more examples

High order bit is the sign bit, otherwise just like unsigned conversion. 4-bit and 8-bit examples:

0110
1110

00001010

11111111

What is the 8-bit, two's complement representation for -7?

For negative values:

1. convert the equivalent positive value to binary
2. then negate binary to get the negative representation

A. 11111001

B. 00000111

C. 11111000

D. 11110011

Negation Two Ways