The Probabilistic Method

Week 3: Asymptotic Analysis, Common Functions

Joshua Brody
CS49/Math59
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Clicker Question

Let **$f(n) = O(g(n))$** such that **$g(n) = \omega(1)$**. Which of the following statements must hold?

(A) **$\log(f(n)) = O(\log(g(n)))$**

(B) **$f(n) = O(g(n))$**

(C) **$2^{f(n)} = O(2^{g(n)})$**

(D) **(A) and (B)**

(E) **(B) and (C)**

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Log, Exponent Rules

Facts: Let **f, g** be functions such that **f = O(g)**.
Then, the following must hold:

(1) Polynomial Rule. $f^k = O(g^k)$ for any integer $k > 1$.

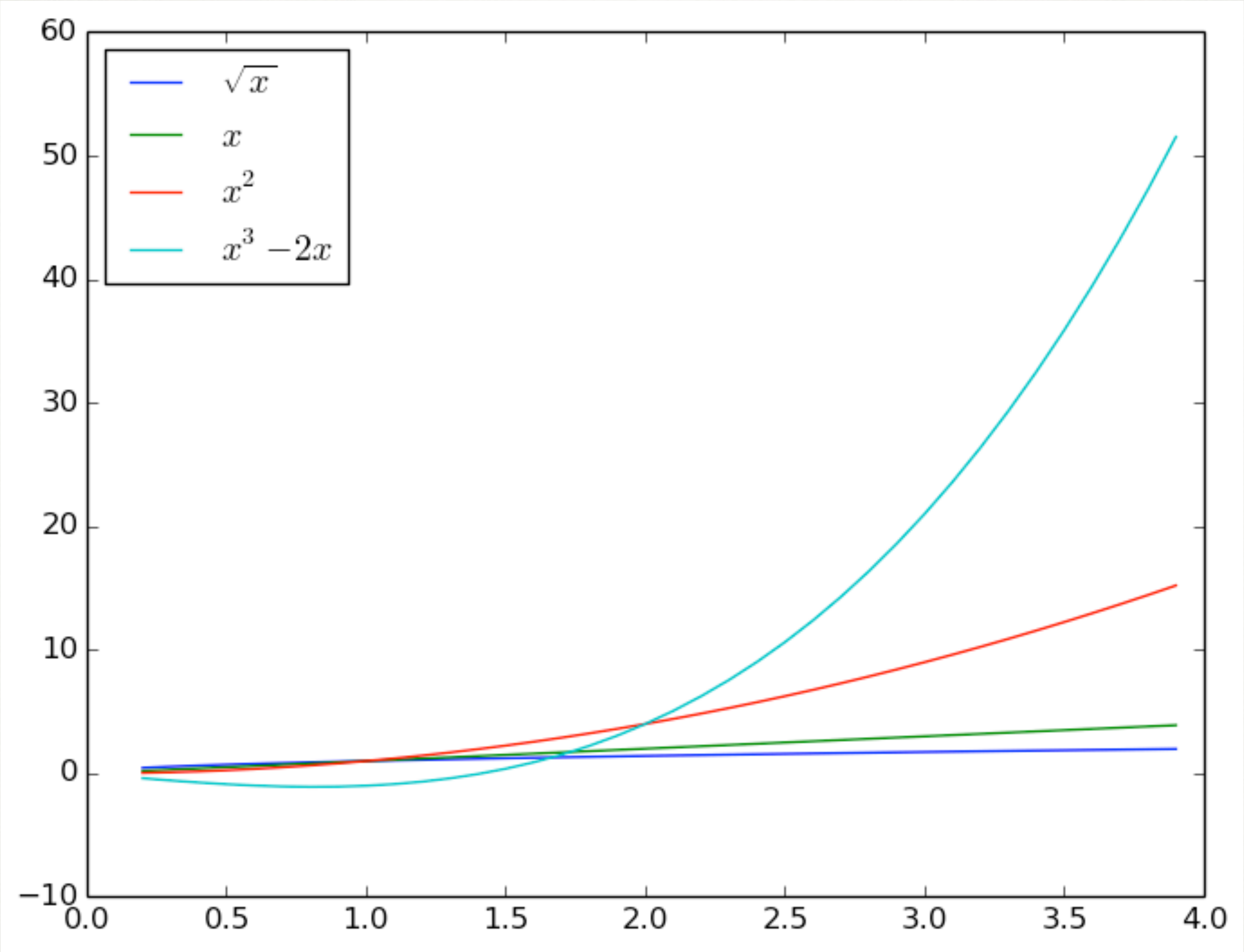
(2) Log Rule. If $g(n) = \omega(1)$ then

$$\log(f(n)) = O(\log(g(n)))$$

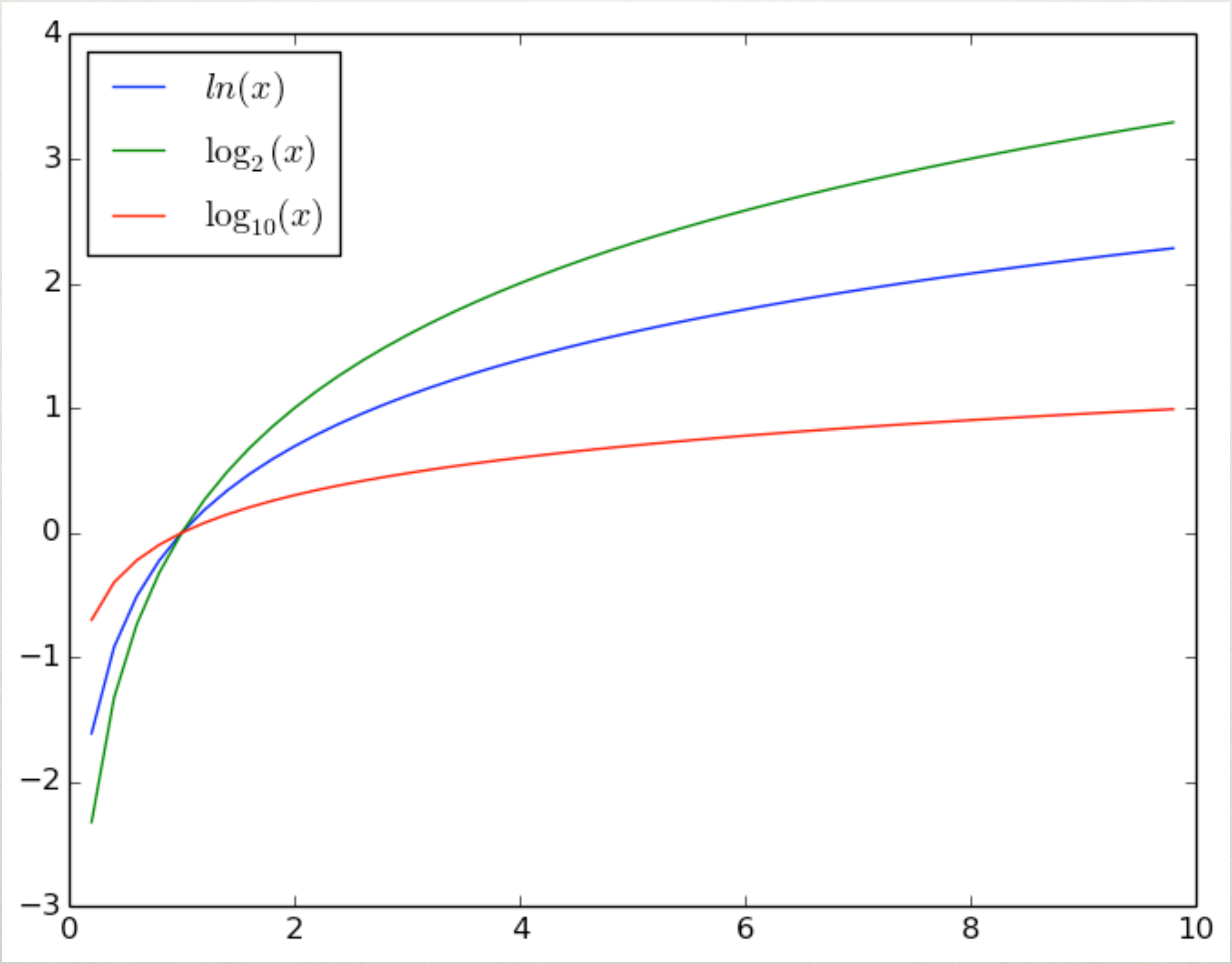
(3) Exponential Rule. If $g(n) = f(n) + \omega(1)$, then

$$2^{f(n)} = o(2^{g(n)})$$

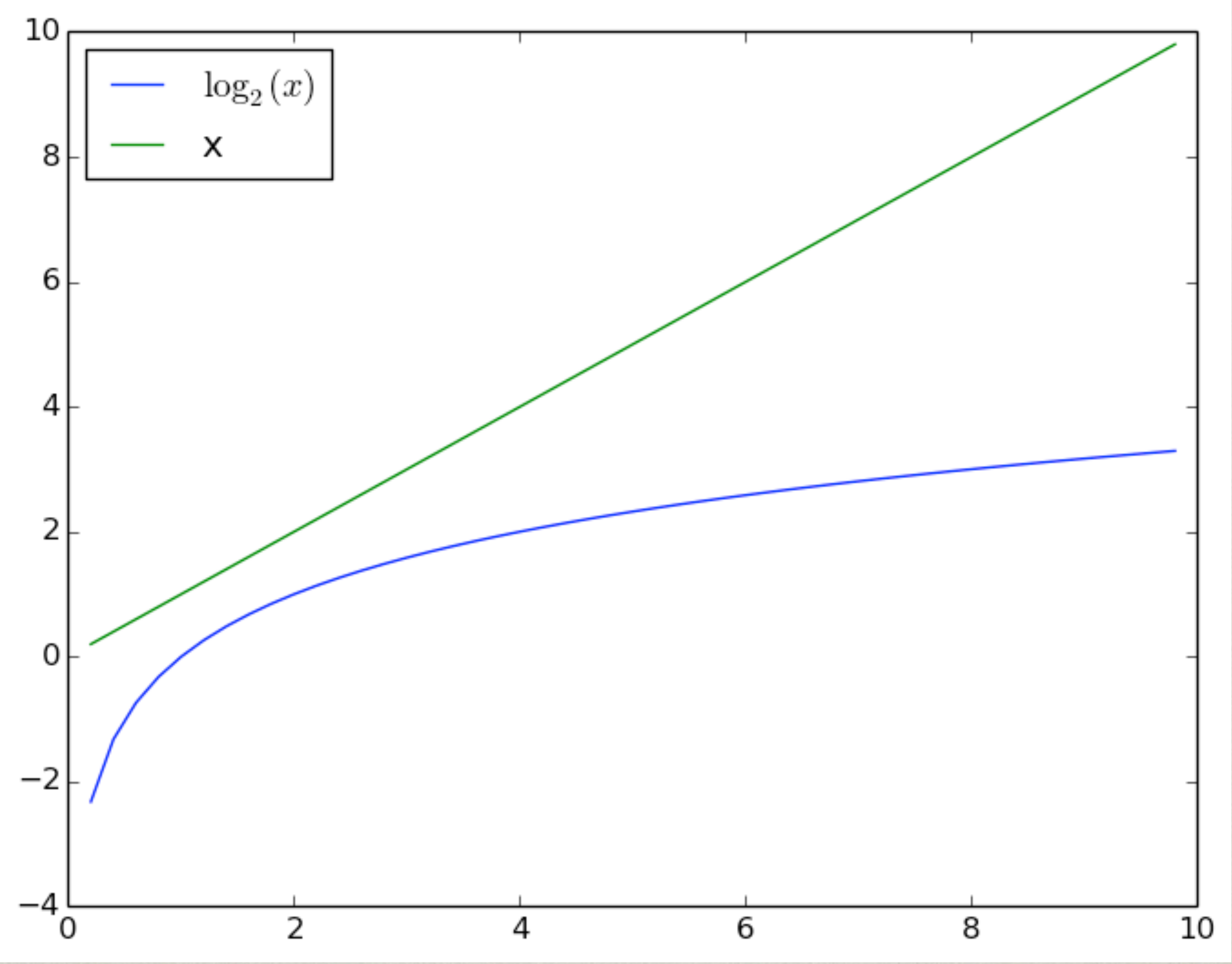
Polynomials



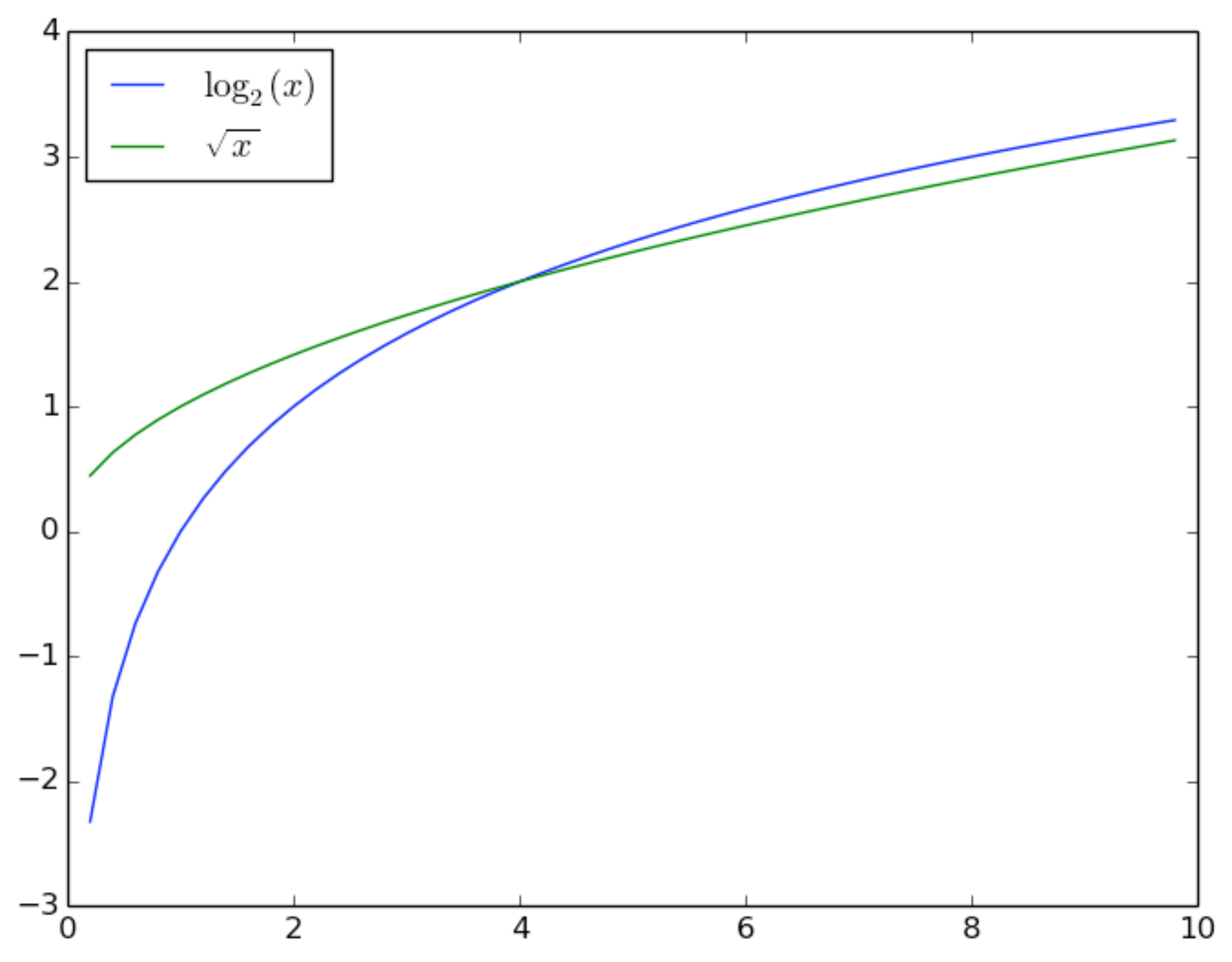
Logarithms



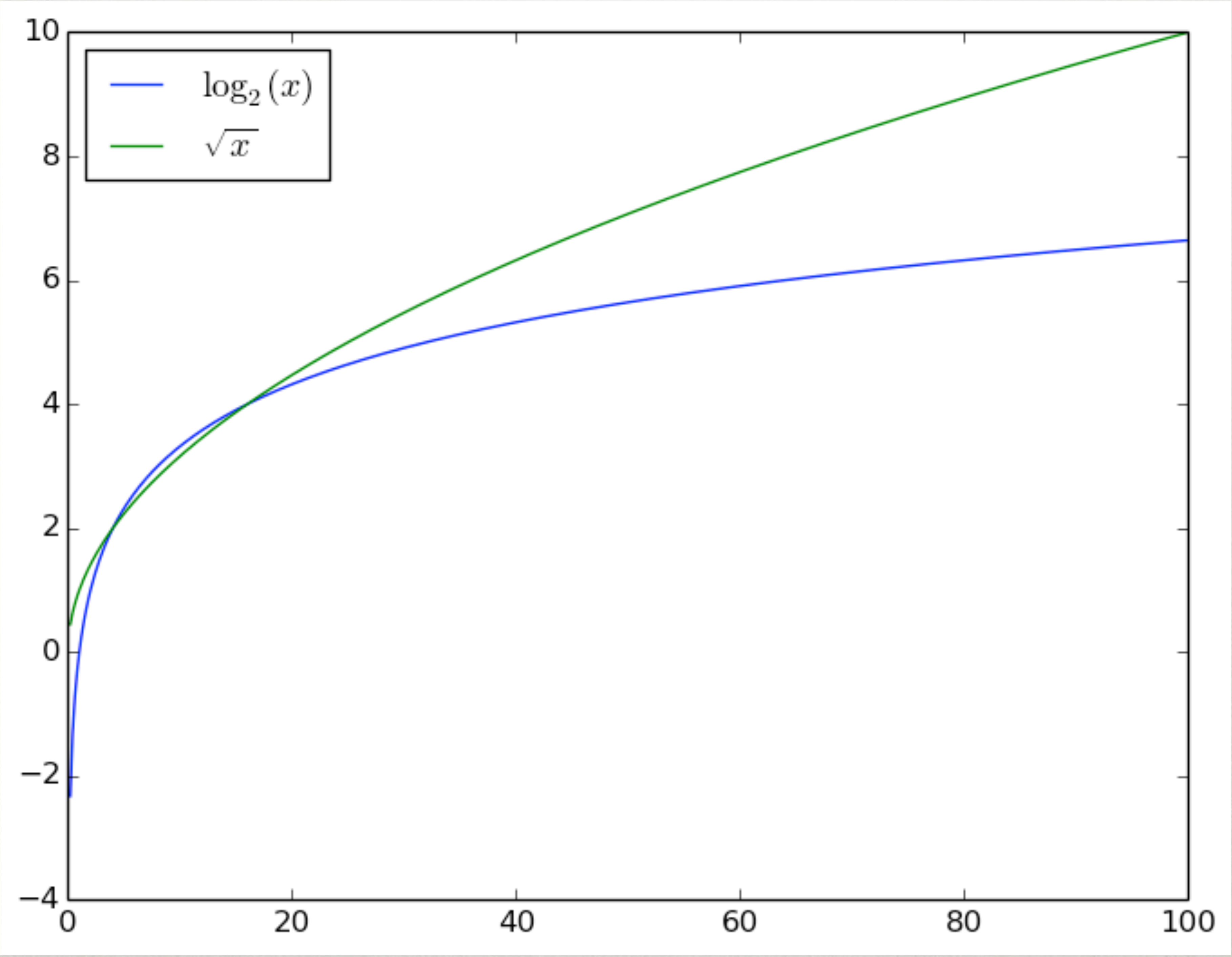
Logarithms vs Polynomials



Logarithms vs Polynomials



Logarithms vs Polynomials



Clicker Question

Let **$1 < s < r$**. Let **$f(n) = r^n$** and **$g(n) = s^n$**. What is the most accurate comparison for **$f(n)$** vs **$g(n)$**?

(A) **$f = O(g)$**

(B) **$f = \Omega(g)$**

(C) **$f = \Theta(g)$**

(D) **$f = o(g)$**

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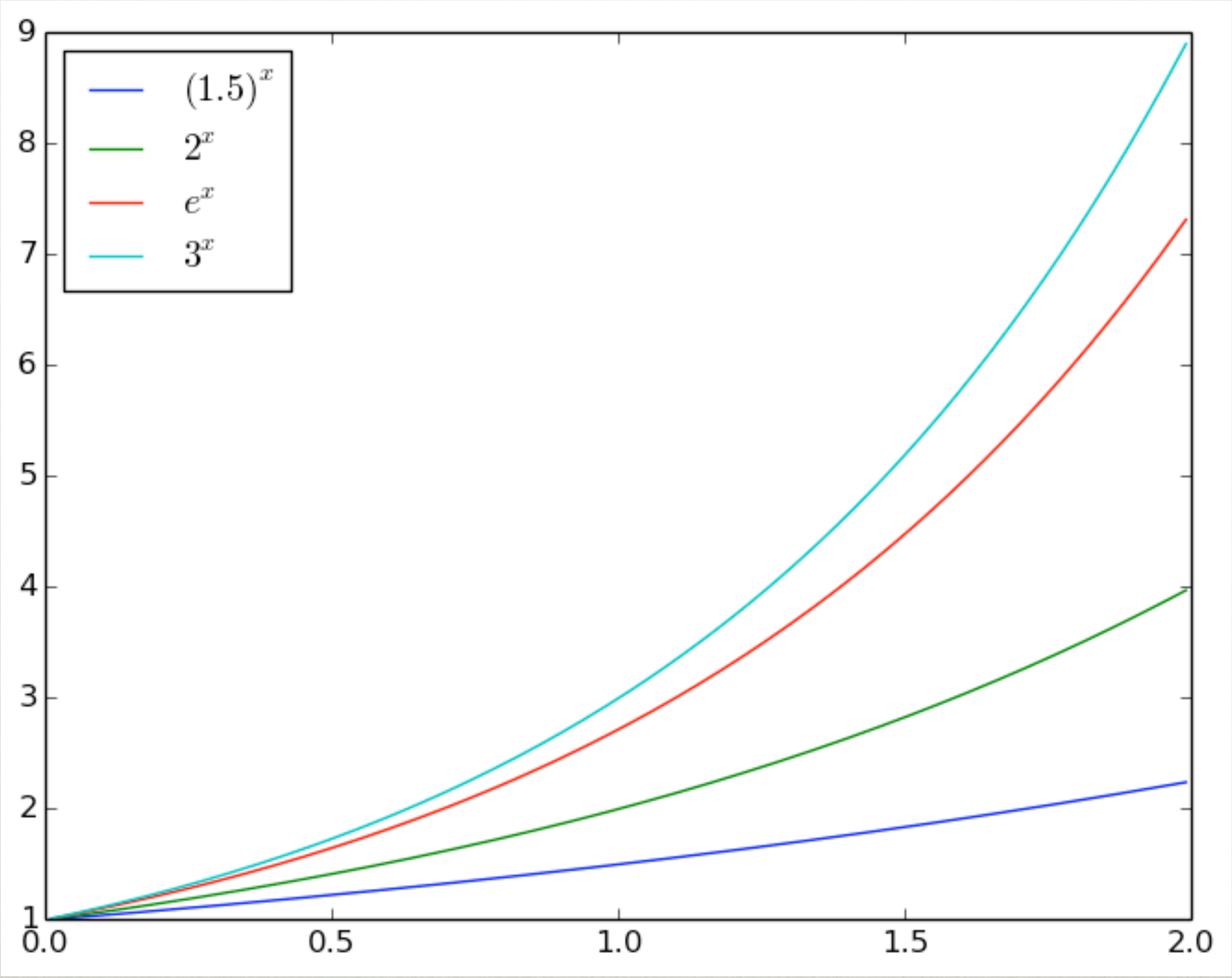
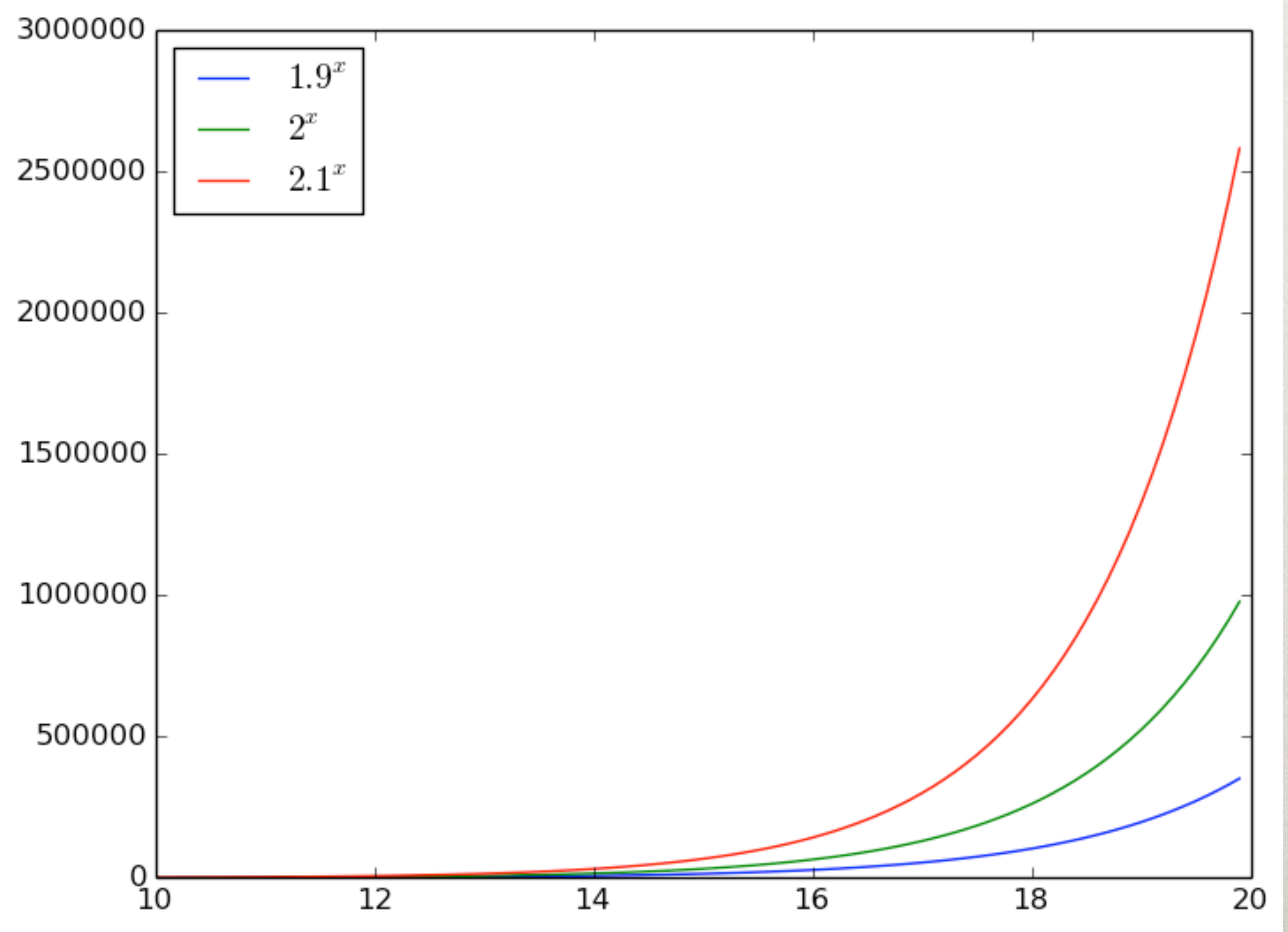
Exponentials



Exponentials



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