The Probabilistic Method

Week 2: Independence, Random Variables, Asymptopia

Joshua Brody
CS49/Math59
Fall 2015

Clicker Question

Suppose that **$f(n) = n^2/10$** and **$g(n) = 100n$**. Which of the following hold?

(A) **$f(n) = O(g(n))$**

(B) **$f(n) = \Omega(g(n))$**

(C) **$f(n) = \Theta(g(n))$**

(D) **A and B**

(E) **B and C**

Clicker Question

Suppose that **$f(n) = n^2/10$** and **$g(n) = 100n$**. Which of the following hold?

(A) **$f(n) = O(g(n))$**

(B) **$f(n) = \Omega(g(n))$**

(C) **$f(n) = \Theta(g(n))$**

(D) **A and B**

(E) **B and C**

Clicker Question

Let **$f(n) = n^2/10$**, **$g(n) = 100n$**, and **$h(n) = n/\log(n)$**.

Order f,g,h in increasing rates of growth.

(A) **f, h, g**

(B) **f, g, h**

(C) **g, f, h**

(D) **g, h, f**

(E) **h, g, f**

Clicker Question

Let **$f(n) = n^2/10$**, **$g(n) = 100n$**, and **$h(n) = n/\log(n)$**.

Order f,g,h in increasing rates of growth.

(A) **f, h, g**

(B) **f, g, h**

(C) **g, f, h**

(D) **g, h, f**

(E) **h, g, f**

Asymptotic Notation

- $f(n) = O(g(n))$ if there exists constants $c, n_0 > 0$ such that for all $n \geq n_0$,

$$f(n) \leq cg(n)$$

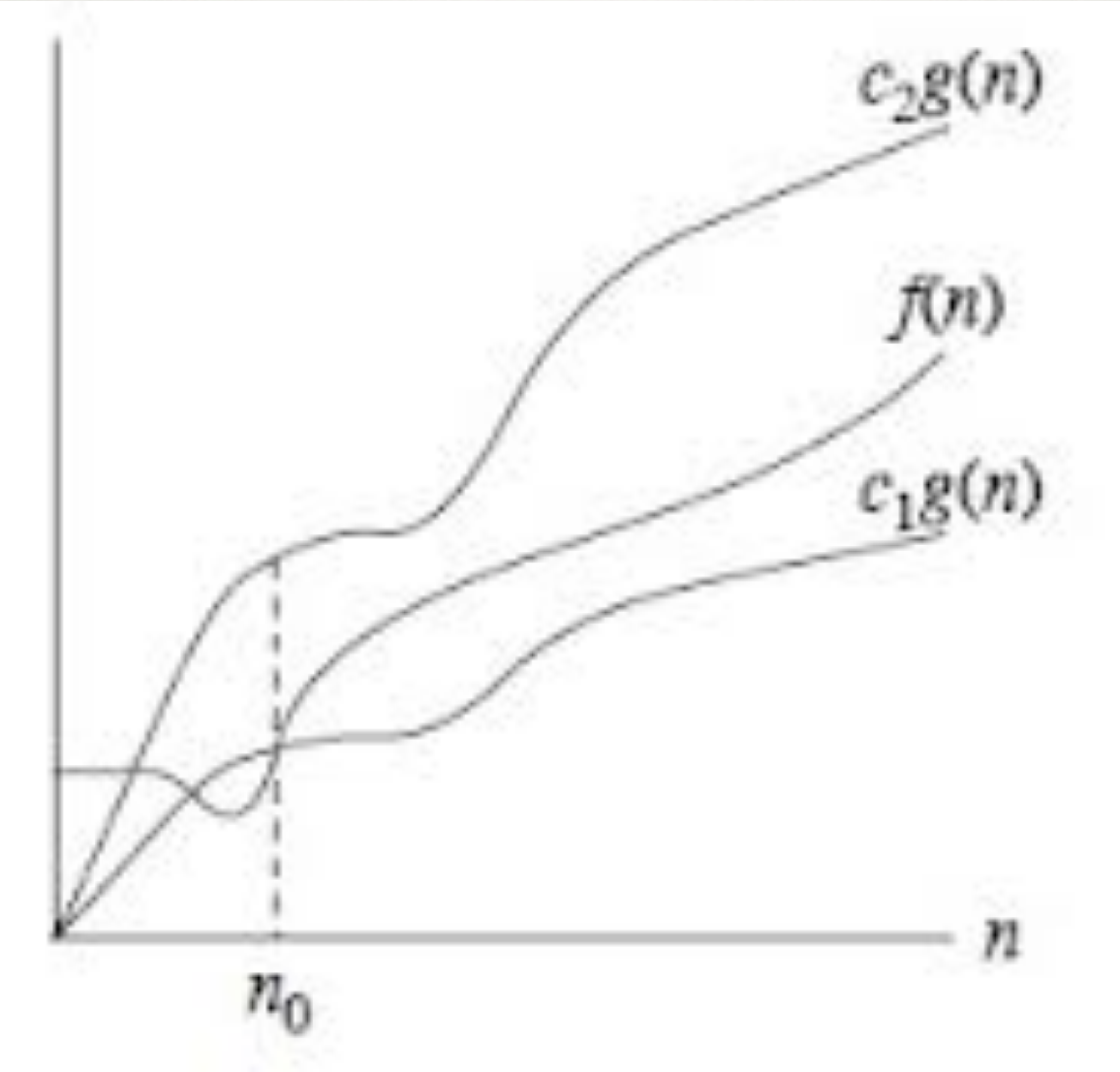
- $f(n) = \Omega(g(n))$ if there exists constants $c, n_0 > 0$ such that for all $n \geq n_0$,

$$f(n) \geq cg(n)$$

- $f(n) = \Theta(g(n))$ if there are constants $c_1, c_2, n_0 > 0$ such that for all $n \geq n_0$,

$$c_1 g(n) \leq f(n) \leq c_2 g(n)$$

Asymptotic Notation



Asymptotic Notation

- **$f(n) = O(g(n))$** if there exists constants **$c, n_0 > 0$** such that for all **$n \geq n_0$**,

$$f(n) \leq cg(n)$$

- **$f(n) = \Omega(g(n))$** if there exists constants **$c, n_0 > 0$** such that for all **$n \geq n_0$**,

$$f(n) \geq cg(n)$$

- **$f(n) = \Theta(g(n))$** if there are constants **$c_1, c_2, n_0 > 0$** such that for all **$n \geq n_0$**,

$$c_1 g(n) \leq f(n) \leq c_2 g(n)$$

Clicker Question

Let **$f(n) = 15n+3$**, and **$g(n) = n^2/10$**.

Which values for **(c,n_0)** would be valid choices for showing **$f = O(g)$**?

(A) **$(c,n_0) = (10, -1)$**

(B) **$(c,n_0) = (1, 10)$**

(C) **$(c,n_0) = (4, 4)$**

(D) **$(c,n_0) = (60, 3)$**

(E) **$(c,n_0) = (100, 1)$**

Clicker Question

Let **$f(n) = 15n+3$**, and **$g(n) = n^2/10$**.

Which values for **(c,n_0)** would be valid choices for showing **$f = O(g)$**?

(A) **$(c,n_0) = (10, -1)$**

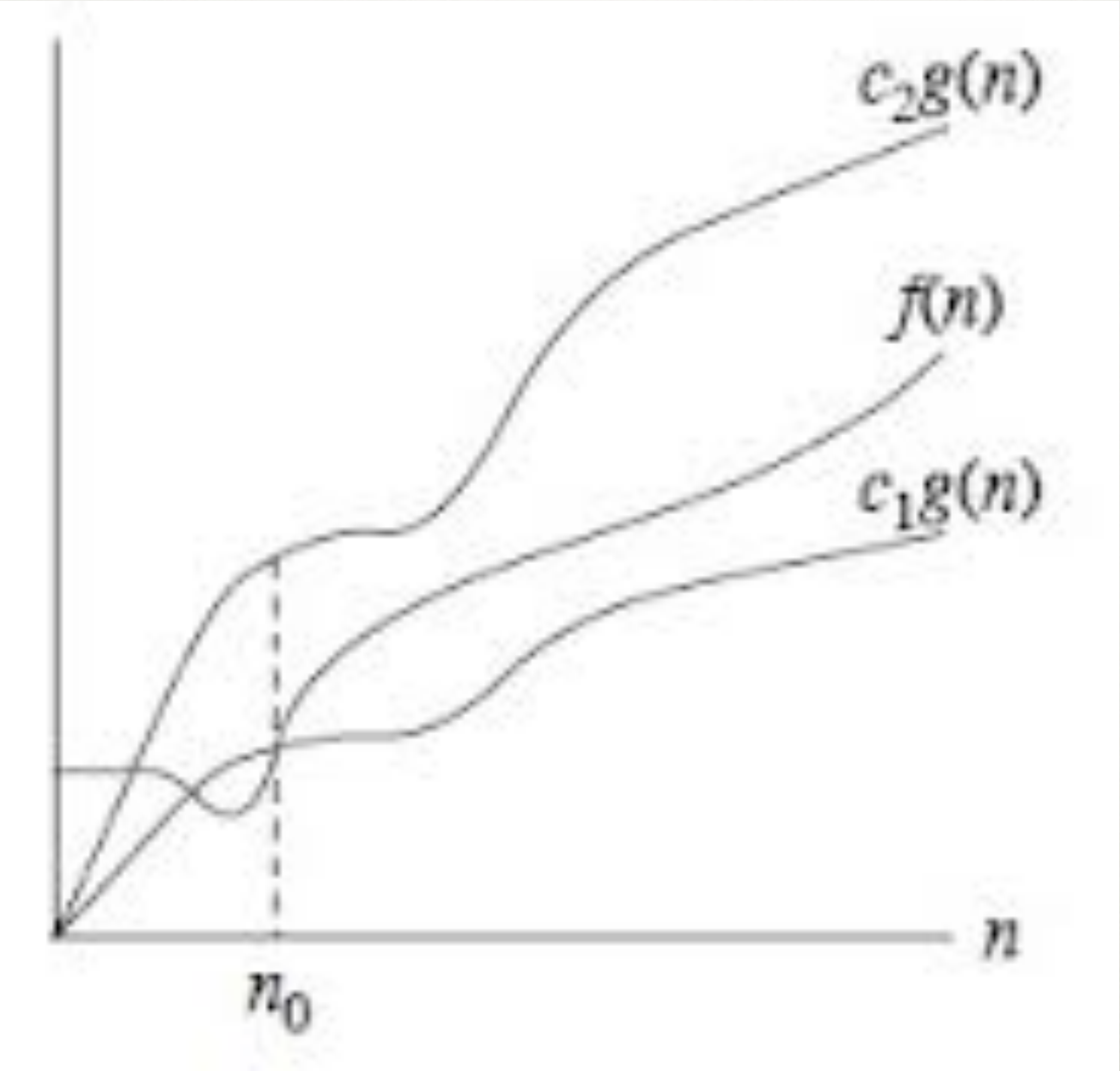
(B) **$(c,n_0) = (1, 10)$**

(C) **$(c,n_0) = (4, 4)$**

(D) **$(c,n_0) = (60, 3)$**

(E) **$(c,n_0) = (100, 1)$**

Asymptotic Notation



Alternate Facts/ Definitions

Facts:

(1) $f(n) = \Theta(g(n))$ **iff** $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$

(2) $f(n) = o(g(n))$ **iff** $f(n) = O(g(n))$ but not $f(n) = \Omega(g(n))$

(3) $f(n) = w(g(n))$ **iff** $f(n) = \Omega(g(n))$ but not $f(n) = O(g(n))$

Limit-based definitions: if $\lim_{n\to\infty} f(n)/g(n)$ exists:

(1) $f(n) = \Theta(g(n))$ **iff** $\lim_{n\to\infty} f(n)/g(n) = c$ for some $c>0$

(2) $f(n) = o(g(n))$ **iff** $\lim_{n\to\infty} f(n)/g(n) = 0$

(3) $f(n) = w(g(n))$ **iff** $\lim_{n\to\infty} f(n)/g(n) = \infty$

(4) $f \sim g$ **iff** $\lim_{n\to\infty} f(n)/g(n) = 1$

The Probabilistic Method