The Probabilistic Method

Week 11: Applications, Randomized Algorithms

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CS49/Math59

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Reading Quiz

What resource is typically not an important factor in the analysis of streaming algorithms?

(A) memory space (hint: not the right answer)

(B) runtime

(C) power consumption

(D) approximation factor

(E) error guarantee

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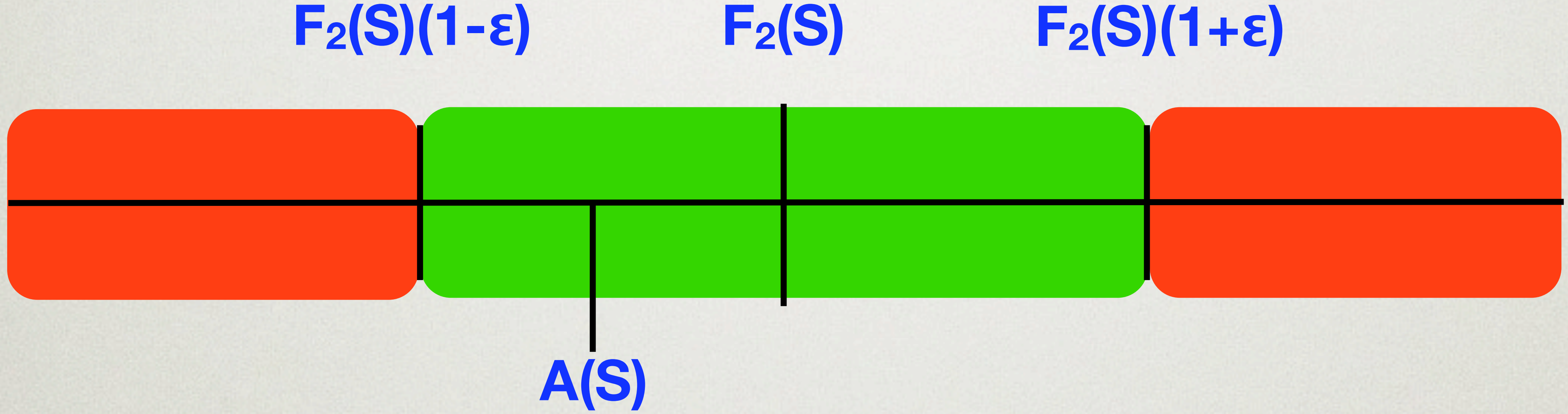
PAC Algorithms

Probably Approximately Correct (PAC) model:

"most of the time we're close enough"



Definition: A streaming algorithm **A** is an (ε,δ)-approximation for **f** if for any stream **S**

$$\Pr[f(s)(1-\varepsilon) \leq A(S) \leq f(s)(1+\varepsilon)] \geq 1-\delta$$

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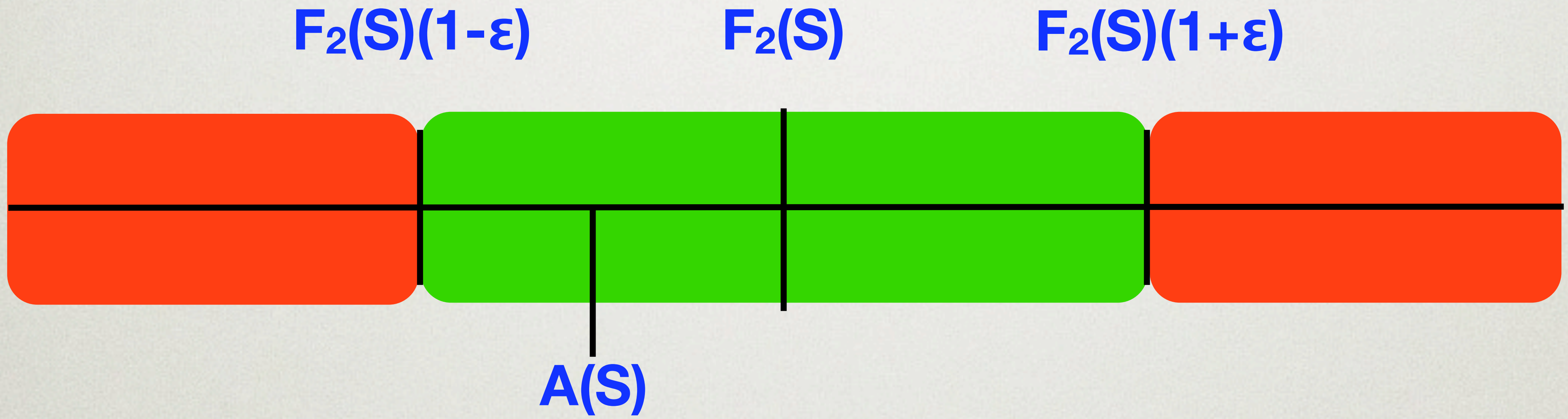

Theorem: For any $\varepsilon > 0$, there is a streaming algorithm that $(\varepsilon, 1/3)$-approximates for F_2 in $O([\log(n) + \log(m)]/\varepsilon^2)$ space.

Clicker Question

For what values of ε does the AMS Basic Estimator have error **< 1/3**?

(A) no ε makes this work

(B) ε > 1/3

(C) ε > π/2

(D) ε > √6

(E) None of the above

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Clicker Question

For arbitrary $\varepsilon > 0$, what values of k give error $< 1/3$?

(A) no k makes this work

(B) $k > 6/\varepsilon$

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For arbitrary $\varepsilon > 0$, what values of **k** give error **< 1/3**?

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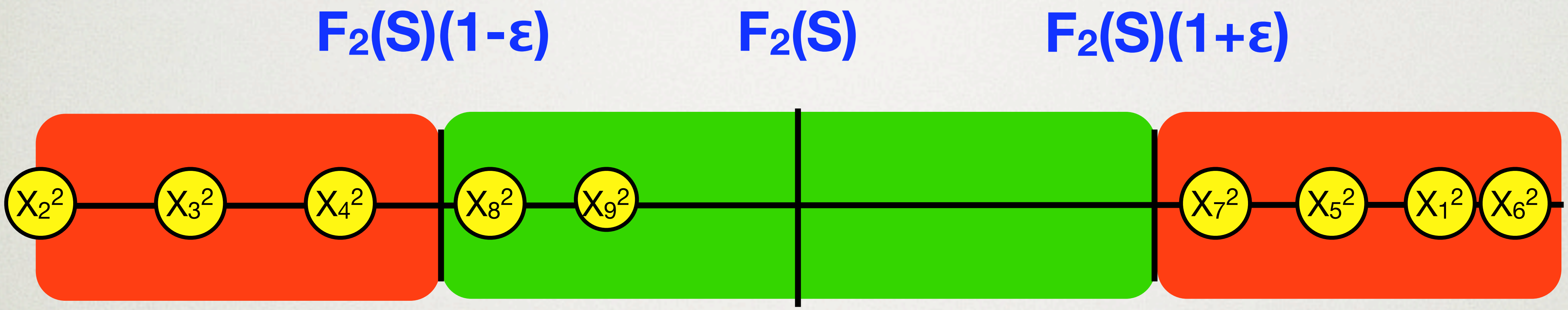
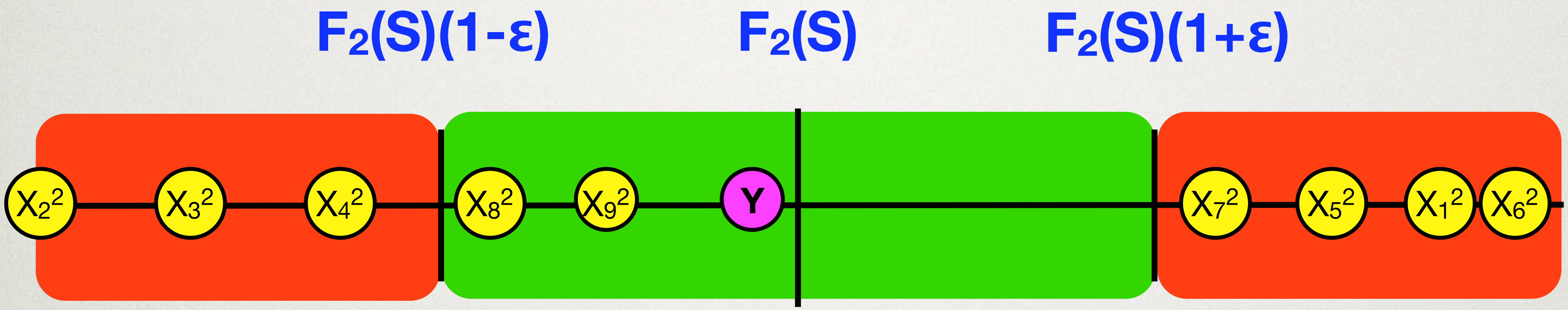
Improved F_2 Estimator



AMS Improved Estimator **$Y := (\sum X_i^2)/k$**

Improved F_2 Estimator



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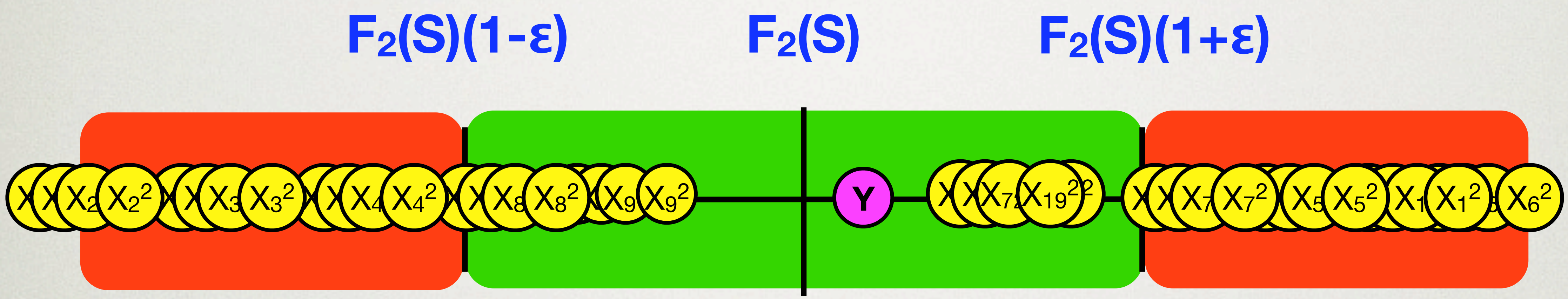

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- **$k = \Omega(1/\delta\varepsilon^2)$** : error **< δ**

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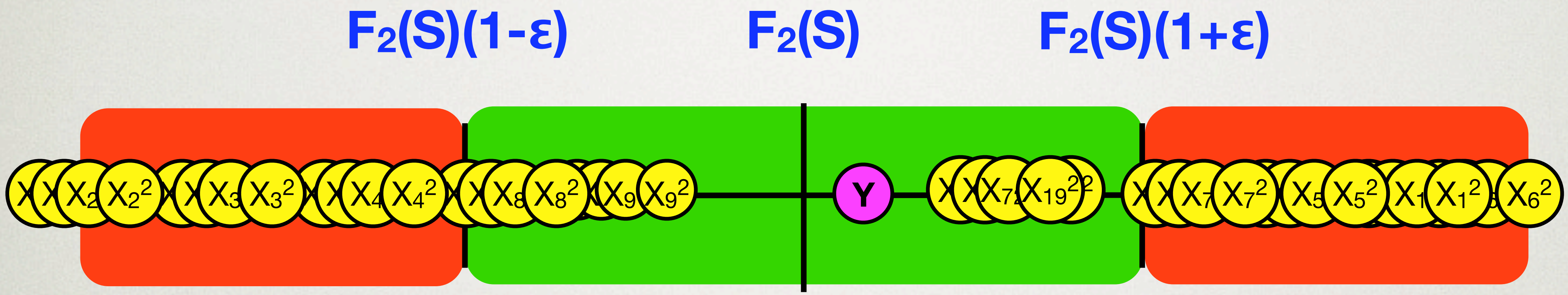

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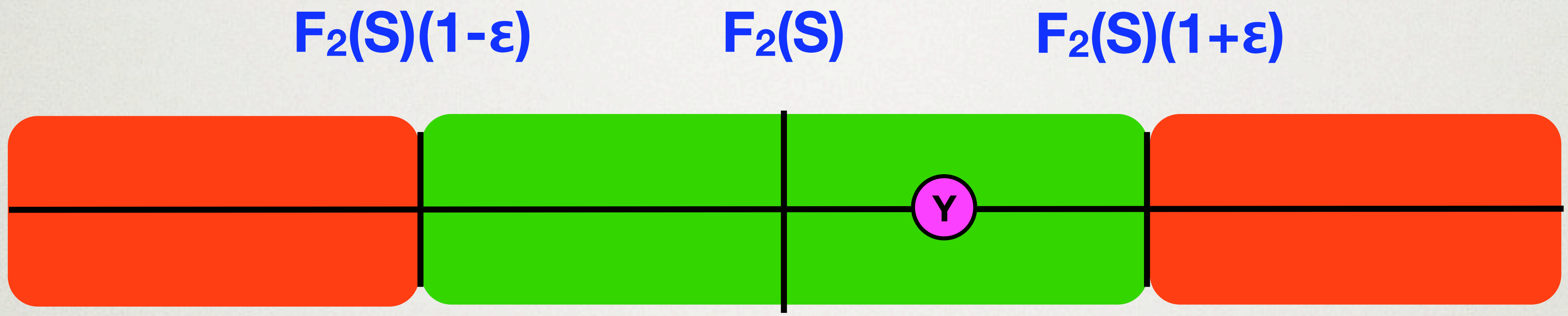
Better error dependence possible with Chernoff Bounds

Final F_2 Estimator



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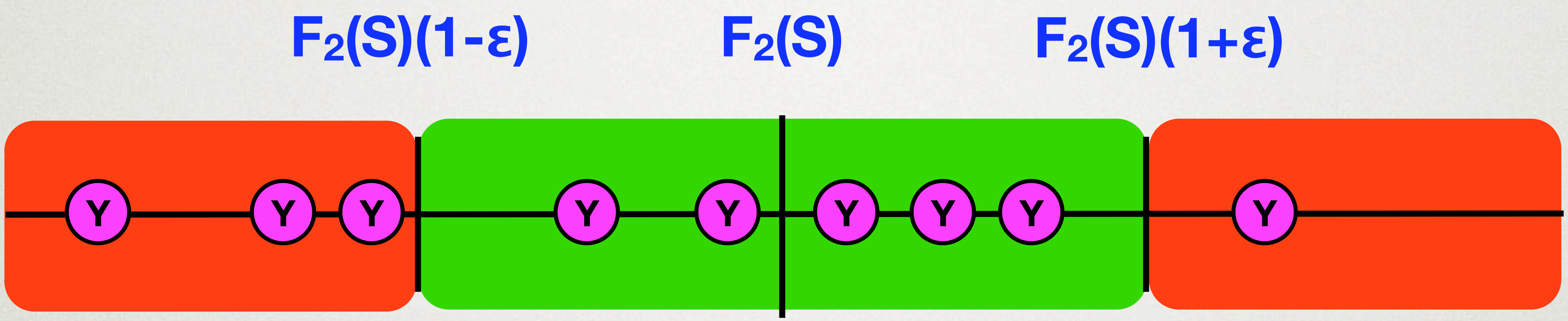

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- **$k' = \Omega(\log(1/\delta))$** : error **$< \delta$**

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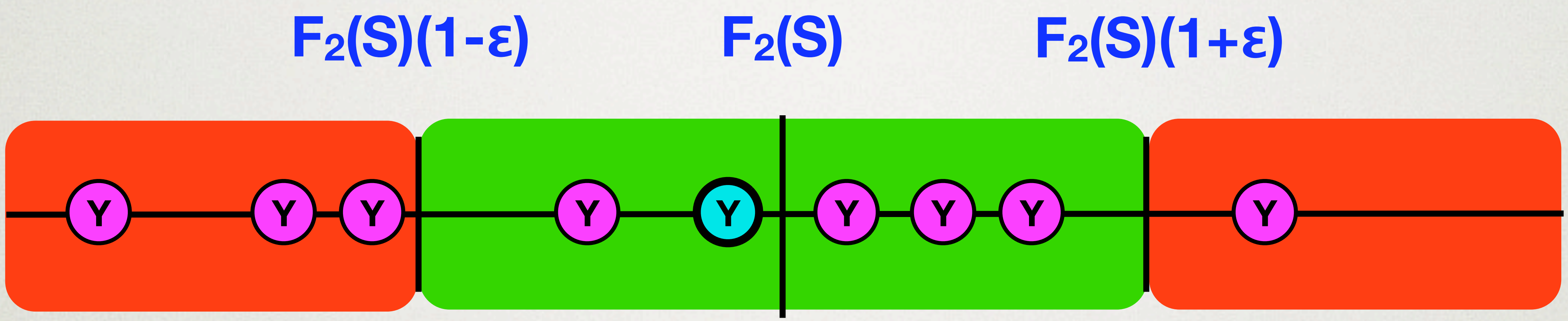

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