

THE PROBABILISTIC METHOD

WEEK 1: INTRODUCTION TO PROBABILITY THEORY



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CS49/MATH59
FALL 2015

CLICKER QUESTION

What is a probability distribution?

- (A) A probability distribution is a **random variable**.
- (B) A probability distribution is a **function**.
- (C) A probability distribution is an **event**.
- (D) **None of the above**.

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PROBABILITY DISTRIBUTIONS

Let Ω be a finite, non-empty set. A probability distribution on Ω is a function $\mathbf{P}: \Omega \rightarrow \{0, 1\}$ such that

$$\sum_{\mathbf{w} \in \Omega} \mathbf{P}(\mathbf{w}) = 1$$

- Ω : sample space of \mathbf{P}
- support of \mathbf{P} : $\{\mathbf{w} : \mathbf{P}(\mathbf{w}) > 0\}$

CLICKER QUESTION

binomial distribution: flip a coin n times and count # heads.

What is **Pr[k]**?

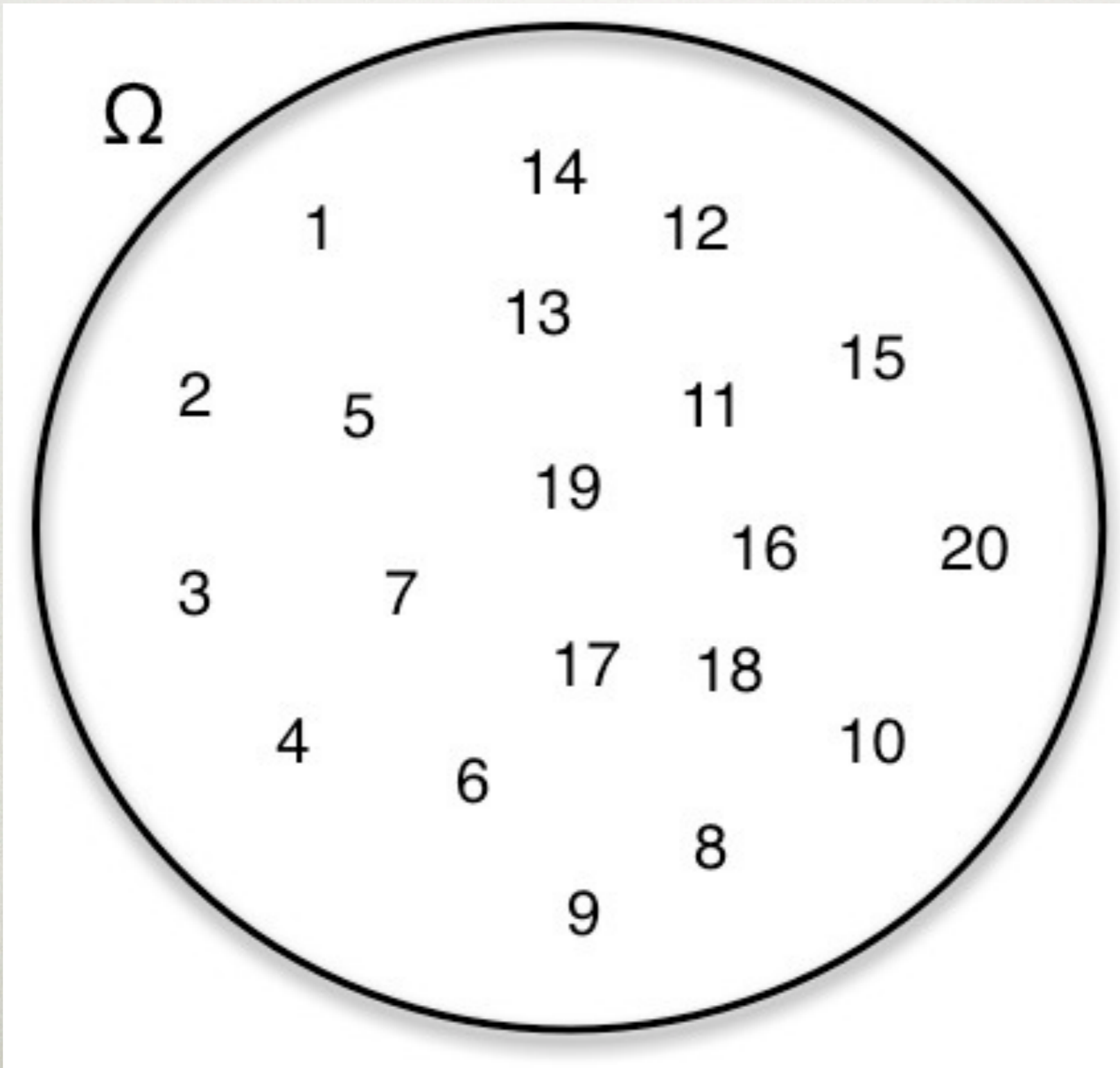
(A) **Pr[k] = $1/n$.**

(B) **Pr[k] = k/n .**

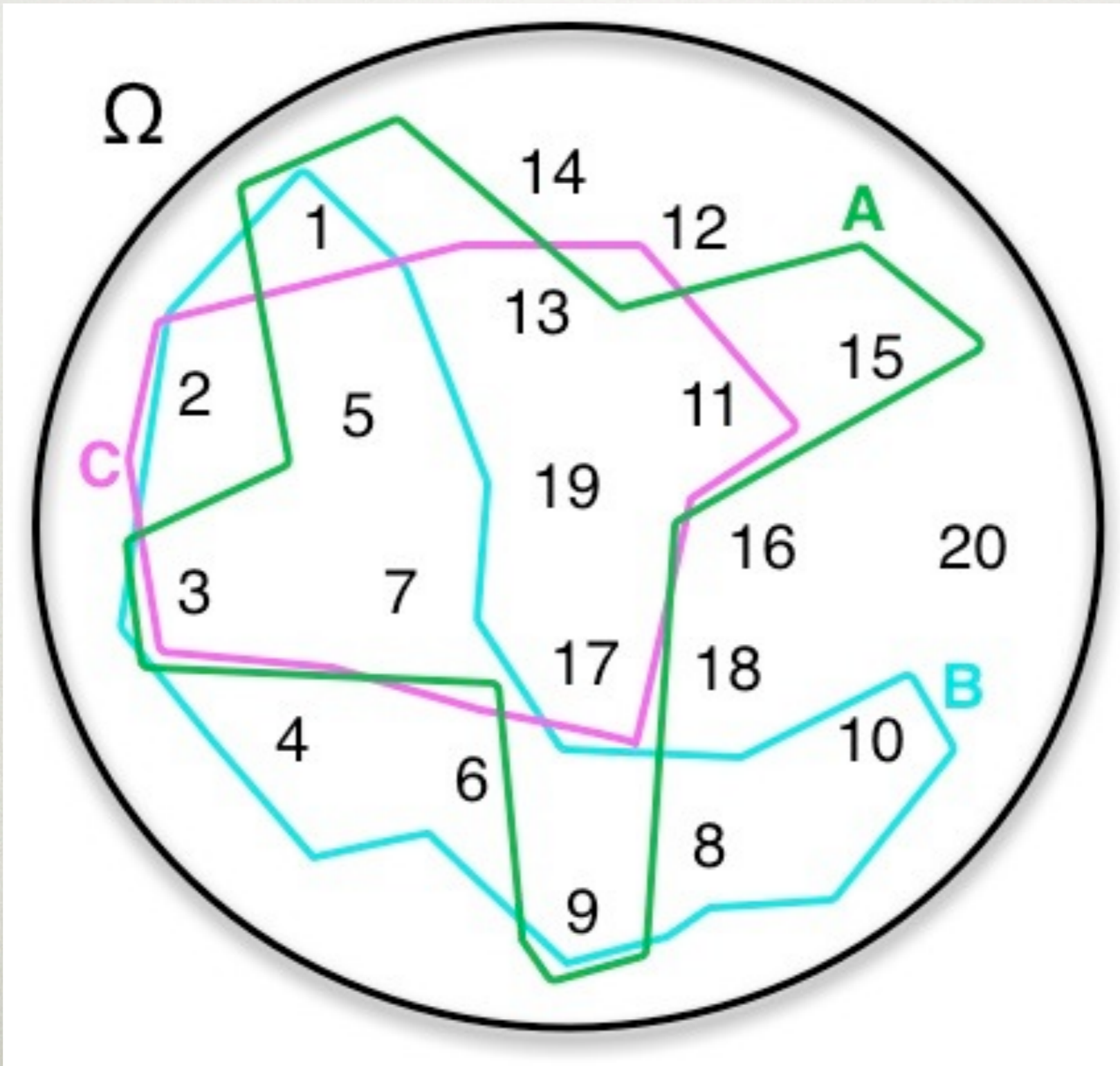
(C) **Pr[k] = $\binom{n}{k} / 2^n$.**

(D) **Pr[k] = $k^2 / 2^n$.**

EVENTS



EVENTS



EXAMPLE PROBLEM

Arithmetic Sequence:

list of numbers (a_1, a_2, \dots, a_m) where $a_i = a_{i-1} + k$ for some k

examples: $(1, 5, 9, 13)$, $(201, 402, 603, 804, 1005)$

Problem: show how to color numbers $\{1, 2, 3, 4, \dots, 2015\}$ using 4 colors so that no arithmetic sequence is *monochromatic*

Solution: color each number $1, 2, \dots, 2015$ *randomly*.

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