CS46, Swarthmore College, Spring 2018
Lab 7 (due Wednesday 4 April)
Name: YOUR NAME(S) HERE

1. Recall that every Turing-recognizable language R has an enumerator E that enumerators it.

   (a) (Sipser 3.19) Show that every infinite Turing-recognizable language R has an infinite decidable subset D.

   (b) Is it possible for R to contain only a finite number of pairwise disjoint infinite decidable subsets? Briefly explain your answer.

2. Given a grammar G, we say that a variable $V \in G$ is useless if there is no string w for which a possible derivation of w contains the variable V. Formulate this problem of finding grammars containing useless variables as a language and show that this language is decidable.

3. (Sipser 5.2) Show that EQ_{CFG} is co-Turing-recognizable. I highly recommend non-determinism.

4. Consider the language $L = \{\langle M, w\rangle \mid M$ is a single tape TM that never modifies the portion of the tape that contains the original input $w\}$.

   (a) Show that L is co-Turing-recognizable, by briefly describing the elements of $\overline{L}$ and then describing a recognizer for $\overline{L}$.

   (b) Is L decidable? Prove your answer. Note if you can show that L is Turing-recognizable, you can apply Theorem 4.22 and part (a) to show L is decidable.