In lab exercises

1. Selection sort on a array of n items can be described recursively as follows: find the smallest element of the array, move it to the front, selection sort the rest of the array. Write a recurrence equation that describes the run time of selection sort and then solve the recurrence.

2. Solve
$$T(n) = \begin{cases} 2T(\frac{n}{2}) + c_1 n^2 & \text{if } n > 1 \\ c_2 & n = 1 \end{cases}$$

3. Solve
$$T(n) = \begin{cases} 2T(n-1) + n & \text{if } n > 1 \\ c_2 & n = 1 \end{cases}$$

4. Solve
$$T(n) = \begin{cases} T(n-1) + T(n-2) + 1 & \text{if } n > 2 \\ c_2 & \text{otherwise} \end{cases}$$

5. Solve
$$T(n) = \begin{cases} 2T(\frac{n}{4}) + c_1 & \text{if } n > 4 \\ c_2 & \text{otherwise} \end{cases}$$

6. Solve
$$T(n) = \begin{cases} 2T(\sqrt{n}) + \lg n & \text{if } n > 2 \\ c_2 & \text{otherwise} \end{cases}$$

   This one is a bit evil. Start by letting $m = \lg n$. Rewrite the recurrence in terms of m. Now let $S(m) = T(2^m)$ and rewrite the recurrence in terms of $S(m)$. This recurrence should look familiar, solve it. Finally express your answer in terms of T and n.

7. **Liars and Friars** For fall break, you escape to a tropical island to get away from Philly weather and Algorithms class. The island is populated by n inhabitants, where each inhabitant is either a liar or a friar. A friar always tells the truth, but liars cannot be trusted. You want to find the best place to eat for dinner and you certainly don't want to ask a liar (they will recommend bagel bar at Sharples, or some hipster coffee shop), so you would like to identify at least one friar. To help find a friar, you can pair up any two inhabitants A and B and ask each to identify the other. Each answers that the other is either a liar or a friar. If either A or B answers that the other is a liar, at least one of A and B is a liar. If both claim that the other is friar, A and B are either both friars or both liars.

   (a) Show if more than $n/2$ inhabitants are liars, you may not be able to identify a friar using this pairwise strategy. You may assume the liars can collude to convince you they are friars.

   (b) Now assume that more than $n/2$ inhabitants are friars. Show that $\lfloor n/2 \rfloor$ pairwise comparisons can reduce the problem of finding a single friar to a problem of nearly half the size. Describe how to find a single friar using this approach.

   (c) Show how to find all friars, assuming there are more than $n/2$ using no more than $O(n)$ pairwise comparisons. Give a recurrence which counts the number of comparisons and give a solution to this recurrence.