In lab exercises

1. We can show 2-SAT is in P using the following ideas. Consider a clause $(x_1 \vee x_2)$. We can view this as two implications $\overline{x_1} \rightarrow x_2$ and $\overline{x_2} \rightarrow x_1$. Now consider the implication $u \rightarrow v$ as a directed edge in graph from u to v. We call this graph the implication graph. The general claim is that a 2-SAT formula is satisfiable iff there is no path in the implication graph from x_i to $\overline{x_i}$ for any i. Furthermore, we can compute a truth assignment for all x_i as follows.

   (a) build the implication graph
   (b) identify all strongly connected components in the implication graph
   (c) if any SCC has both x_i and $\overline{x_i}$, then the formula is not satisfiable
   (d) Construct the SCC graph where each vertex in the SCC graph is a single SCC and there is and edge from u to v in the SCC graph if there is an edge from one term in the SCC of u to a term in the SCC of v.
   (e) Topologically order the SCC graph
   (f) For each component in topo order, if its terms do not already have truth assignments, set all the terms in the component to be false. This may cause you to set the negation of all these terms to true in other components

   Given this algorithm, complete the following

   (a) Apply the algorithm to the formula $F_1 = (\overline{x_0} \vee x_1) \wedge (\overline{x_1} \vee x_2) \wedge (x_0 \vee \overline{x_2}) \wedge (x_2 \vee x_1)$. Give a satisfiable assignment for F_1 if one exists, or show that no satisfiable assignment exists.
   (b) Repeat for $F_2 = F_1 \wedge (\overline{x_3} \vee x_4) \wedge (\overline{x_4} \vee x_3) \wedge (\overline{x_1} \vee x_3)$
   (c) Repeat for $F_3 = F_1 \wedge (\overline{x_0} \vee \overline{x_2})$
   (d) Why does the algorithm work?
   (e) What is the runtime of your algorithm?

2. Show SATISFIABILITY $\leq_P$ 3-SAT. Don't worry about short clauses. We fixed that in the CIRCUIT-SAT $\leq_P$ 3-SAT reduction. Consider a long clause $(x_1 \vee x_2 \vee \ldots x_k)$ and describe how to break it into an equivalent conjunction of 3-clauses. Hint, you will need to add $k-3$ new terms $y_1, y_2, \ldots y_{k-3}$. Try to break up a 4-clause first, then generalize.

3. We know 3-SAT is NPC. What does the previous reduction show? If it does not show that SATISFIABILITY is NPC, can you find another way to show SATISFIABILITY is NPC?

4. Now show SATISFIABILITY $\leq_P$ 2-SAT.