

Homework 7. You may work with one partner on this assignment. Writeups are due Thursday 6 December.

1. For each of the questions below answer “yes”, “no”, or “unknown, because it would result in deciding if $P=NP$ ”. If the answer is “no” or “unknown”, give a brief explanation. If the answer is “yes”, give a reduction.
 - (a) Let DIS be the following problem: Given a collection of intervals $(a, b), a < b$ on a time-line, and a bound k , does the collection contain a subset of at least k non-overlapping intervals? This is a decision problem version of the Interval Scheduling problem we discussed in the greedy algorithms chapter. Question: Is $DIS \leq_P$ Vertex Cover?
 - (b) Is Independent Set \leq_P DIS?
2. Read the definition of the Set Packing problem on page 458. Show Independent Set \leq_P Set Packing.
3. The 2-Color problem asks if each vertex in a graph $G = (V, E)$ can be colored with one of two colors; red or blue such that for every edge has end points of opposite colors.
 - (a) Give an example of a graph that can be 2-Colored
 - (b) Give an example of a graph that cannot be 2-Colored
 - (c) Give an example certificate to show 2-Color is in NP
 - (d) Give an algorithm to show 2-Color is in P: Hint the Oracle of Bacon graph from cs35 can be two colored if edges connect movie/actor pairs and movies are assigned one color and actors/actresses are assigned the other color.
 - (e) 2-SAT is a version of SATISFIABILITY in which each clause has exactly two terms. Let the Boolean term v_{ir} have the value 1 if $v_i \in G$ is colored red, and let the Boolean term v_{ib} have the value 1 if $v_i \in G$ is blue. Give a reduction 2-Color \leq_P 2-SAT. Hint: write 2-SAT clauses that show the following
 - i. Each vertex $v_i \in G$ has at least one color; red or blue
 - ii. Each vertex has at most one color
 - iii. For each edge $(v_i, v_j) \in E$, v_i and v_j have different colors.Remember to show a graph G is 2-Colorable iff your 2-SAT formula is satisfiable.
 - (f) We know 2-Color is in P. What does this imply about 2-SAT?