CS41 lab 4

In typical labs this semester, you'll be working on a number of problems in groups of 3-4 students. You will not be handing in solutions; the primary purpose of these labs is to have a low-pressure space to discuss algorithm design. However, it will be common to have some overlap between lab exercises and homework sets.

The learning goals of this lab session are to gain more experience with directed and undirected graphs, and to practice algorithm design for graph problems. There are more problems on this writeup than you have time to complete. Consider the lab a success if you can make good progress on one problem.

1. **Ethnographers.** (Kleinberg and Tardos, 3.12) You're helping a group of ethnographers analyze some oral history data they've collected by interviewing members of a village to learn about the lives of people who have lived there over the past two hundred years.

   From these interviews, they've learned about a set of n people (all now deceased), whom we'll denote $P_1, P_2, \ldots, P_n$. They've also collected facts about when these people lived relative to one another. Each fact has one of the following two forms:

   - for some i and j, person P_i died before person P_j was born; or
   - for some i and j, the lifespans of P_i and P_j overlapped at least partially.

   Naturally, the ethnographers are not sure that all these facts are correct; memories are not very good, and a lot of this was passed down by word of mouth. So what they'd like you to determine is whether the data they've collected is at least *internally consistent*, in the sense that there could have existed a set of people for which all the facts they've learned simultaneously hold.

   Give an efficient algorithm to do this: either it should propose dates of birth and death for each of the n people so that all the facts hold true, or it should report (correctly) that no such dates can exist—that is, the facts collected by the ethnographers are not internally consistent.

2. **Enemies on the Move.** Alice and Bob are very active students at University of Pennsylvania. They used to be best friends but now despise each other. Alice and Bob can't stand to be in the same room, or even nearby. However, they each take many classes and are active in several clubs. Is it even possible to avoid each other?

   This can be modeled as a graph problem. The input consists of:

   - a directed graph $G = (V, E)$,
   - an integer $k \leq n$,
   - start vertices s_A and $s_B \in V$, and
   - end vertices t_A and $t_B \in V$.

   In this problem, Alice starts at s_A and wants to travel to t_A, while Bob starts at s_B and wants to travel to t_B. At each time step, either Alice or Bob moves along a single edge. (You can assume they move separately.) At all times, Alice and Bob must be at least k edges apart.

Design and analyze a polynomial-time algorithm that determines if Alice and Bob can get where they want to go while maintaining distance.

(**Hint:** Reduce this problem to s-t Connectivity.)

3. **Cell service on the Appalachian Trail.**
The Appalachian Trail is an approximately 2100 mile trail that runs north-south from Maine to Georgia. Recently, several hikers have started to demand cell service along the trail. Other hikers object, assuming that cell phones will ruin the hiking experience. Leaders of the Appalachian Trail Conservatory, who manage the trail, have decided on a compromise – they plan to install cell phone base stations, but only for service at one of the campground areas on the trail. You've been hired to help decide where to place the base stations. Your goal is to place cell phone base stations at certain points on the trail, so that every campground is within five miles of the base stations.

   Give an efficient algorithm that achieves this goal, using as few base stations as possible. Your algorithm's input should be a List of campground locations, and you should output a List of base station locations that covers all campground locations using the minimal number of base stations.