CS41 lab 2

In typical labs this semester, you'll be working on a number of problems in groups of 3-4 students. You will not be handing in solutions; the primary purpose of these labs is to have a low-pressure space to discuss algorithm design, and to gain experience collaborating with others on algorithm design and analysis. However, it will be common to have some overlap between lab exercises and homeworks.

The goal of this lab is to gain more experience with asymptotic analysis and algorithm design. There are more problems than you have time to complete during the lab. Consider it a successful lab session if you can mostly complete two problems.

1. **Asymptotic analysis properties, part 1.** Assume you have functions f, g, and h. For each of the following statements, decide whether you think it is true or false and give a proof or counterexample.

   (a) If f is $O(h)$ and g is $O(h)$, then $f + g$ is $O(h)$.

   (b) If f is $O(h)$ and g is $O(h)$, then $f \cdot g$ is $O(h)$.

   (c) If f is $O(g)$, then g is $\Omega(f)$.

2. **Asymptotic analysis properties, part 2.** Assume you have functions f and g such that $f(n)$ is $O(g(n))$. For each of the following statements, decide whether you think it is true or false and give a proof or counterexample.

   (a) $\log_2(f(n))$ is $O(\log_2(g(n)))$.

   (b) $2^{f(n)}$ is $O(2^{g(n)})$.

   (c) $(f(n))^2$ is $O((g(n))^2)$.

   (d) If $g(n)$ is $O(h(n))$, then $f(n)$ is $O(h(n))$.

3. **Asymptotic analysis.** Arrange the following functions in ascending order of growth rate. That is, if g follows f in your list, then it should be the case that $f = O(g)$.

   - $f_1(n) = n^{2.5}$
   - $f_2(n) = \sqrt{2n}$
   - $f_3(n) = n + 10$
   - $f_4(n) = 10^n$
   - $f_5(n) = 100^n$
   - $f_6(n) = \log_{1.1}(n)\sqrt[3]{n}$
   - $f_7(n) = n^n$
   - $f_8(n) = n^2 \log_2(n)$
   - $f_9(n) = n^{\log_2(n)}$

4. **Complete analysis.** Let $f(n) = 12n^{4/5}$ and $g(n) = n^{3/5}(\log n)^6$. Prove that $g(n) = O(f(n))$. You may use techniques and facts from class and the textbook; your proof should be formal and complete.