CS41 Homework 4

This homework is due at 11:59PM on Friday, October 11. Write your solution using LaTeX. Submit this homework in a file named `hw4.tex` using **github**. This is an **8 point assignment**. This is a partnered homework. You should primarily be discussing this homework with your partner.

It's ok to discuss approaches at a high level. In fact, I encourage you to discuss general strategies. However, you should not reveal specific details of a solution, nor should you show your written solution to anyone else. The only exception to this rule is work you've done with a lab partner/group *while in lab*. In this case, note (in your post-homework survey) who you've worked with and what parts were solved during lab.

The main **learning goals** of this homework are to get more practice with algorithm design, especially as it pertains to graph algorithms and greedy algorithms.

1. **Network Connectivity**(Kleinberg and Tardos, 3.7) Some friends of yours work on wireless networks, and they're currently studying the properties of a network of n mobile devices. As the devices move around (actually, as their human owners move around), they define a graph at any point in time as follows: there is a node representing each of the n devices, and there is an edge between device i and device j if the physical locations of i and j are no more than 200 meters apart. (If so, say that i and j are *in range* of each other).

   They'd like it to be the case that the network of devices is connected at all times, and so they've constrained the motion of the devices to satisfy the following property: at all times, each device i is within 200 meters of at least $n/2$ of the other devices. (Assume that n is an even number.) What they'd like to know is: Does this property by itself guarantee that the network will remain connected?

   Here's a concrete way to formulate the question as a claim about graphs:

   Claim 1. *Let G be a graph on n nodes, where n is an even number. If every node of G has degree at least $n/2$, then G is connected.*

   Decide whether you think the claim is True or False, and give a proof of either the claim or its negation.

2. **Ethnographers.** (Kleinberg and Tardos, 3.12) You're helping a group of ethnographers analyze some oral history data they've collected by interviewing members of a village to learn about the lives of people who have lived there over the past two hundred years.

   From these interviews, they've learned about a set of n people (all now deceased), whom we'll denote $P_1, P_2, \ldots, P_n$. They've also collected facts about when these people lived relative to one another. Each fact has one of the following two forms:

   - for some i and j, person P_i died before person P_j was born; or
   - for some i and j, the lifespans of P_i and P_j overlapped at least partially.

   Naturally, the ethnographers are not sure that all these facts are correct; memories are not very good, and a lot of this was passed down by word of mouth. So what they'd like you to determine is whether the data they've collected is at least *internally consistent*, in the sense that there could have existed a set of people for which all the facts they've learned simultaneously hold.

Give an efficient algorithm to do this: either it should propose dates of birth and death for each of the n people so that all the facts hold true, or it should report (correctly) that no such dates can exist—that is, the facts collected by the ethnographers are not internally consistent.

3. **Cell service on the Appalachian Trail.**
   The Appalachian Trail is an approximately 2100 mile trail that runs north-south from Maine to Georgia. Recently, several hikers have started to demand cell service along the trail. Other hikers object, assuming that cell phones will ruin the hiking experience. Leaders of the Appalachian Trail Conservatory, who manage the trail, have decided on a compromise – they plan to install cell phone base stations, but only for service at one of the campground areas on the trail. You've been hired to help decide where to place the base stations. Your goal is to place cell phone base stations at certain points on the trail, so that every campground is within five miles of the base stations.

   Give an efficient algorithm that achieves this goal, using as few base stations as possible. Your algorithm's input should be a List of campground locations, and you should output a List of base station locations that covers all campground locations using the minimal number of base stations.