CS41 Homework 1

This homework is due at 11:59PM on Sunday, September 15. This is a 10-point homework. Write your solution using LaTeX. Submit this homework using **github**. This is an individual homework. It's ok to discuss approaches at a high level. In fact, you are encouraged to discuss general strategies. However, you should not reveal specific details of a solution, nor should you show your written solution to anyone else. Please ask Joshua if you have any questions about the academic integrity policy.

The main **learning goals** of this lab are to (i) familiarize you with writing in LaTeX, and (ii) to begin to formalize and analyze algorithms.

1. **EdStem.** Log onto the course forum on EdStem, and either ask a question, or respond to an existing post. Don't feel like your question/post has to be about computer science! The goal is just to make sure you're comfortable using the forum.

2. **Algorithm Analysis.** Consider the following algorithm for the Hiking Problem.

   ```
   HIKING()
   1  k = 1.
   2  while you haven't arrived at your friend:
   3        hike k miles north
   4        return to start
   5        hike k miles south
   6        return to start
   7        k = 5k.
   ```

   Describe the distance traveled in HIKING as a function of the initial distance from your friend in the worst case. Express your answer in big-Oh notation. How does this algorithm compare to the algorithms we saw in class?

3. **Find the Stable Matching.** Below is an input to the stable matching problem:

   - 5 hospitals: [Abington, Brandywine, CHOP, Delaware County Memorial (DCM), Einstein Medical Center (EMC)]
   - 5 doctors: [Alice, Bob, Chenye, Dmitri, Eva]
   - Hospital Preferences (in each list, doctors are ordered from most to least preferred, so e.g. Abington's top choice for doctor is Bob, and least preferred doctor is Alice)
     - Abington: [Bob, Eva, Chenye, Dmitri, Alice]
     - Brandywine: [Eva, Bob, Chenye, Alice, Dmitri]
     - CHOP: [Bob, Chenye, Alice, Eva, Dmitri]
     - Delaware County Memorial: [Chenye, Eva, Alice, Bob, Dmitri]
     - Einstein Medical Center: [Eva, Alice, Dmitri, Bob, Chenye]
   - Doctor Preferences (in each list, hospitals are ordered from most to least preferred)
     - Alice: [Abington, Brandywine, CHOP, DCM, EMC]

- Bob: [CHOP, Brandywine, Abington, EMC, DCM]
- Chenye: [CHOP, Brandywine, Abington, DCM, EMC]
- Dmitri: [DCM, Brandywine, CHOP, Abington, EMC]
- Eva: [CHOP, Brandywine, EMC, DCM, Abington]

Give a stable (hospital-doctor) matching for this input.

4. **Stable Matching Runtime.** We showed in class that the Gale-Shapely Algorithm for stable matching terminates after at most n^2 iterations of the while loop.

   (a) For two sets A and B of size n, can a particular list of rankings actually result in a quadratic number of iterations? If so, describe what the rankings would look like. If not, argue why no set of rankings would ever result in a quadratic number of iterations. **Note:** the algorithm need not take exactly n^2 iterations, but *asymptotically* n^2 iterations, meaning $0.1n^2$ would be sufficient to show your claim.

   (b) Can a particular set of rankings result in strictly less than a quadratic number of iterations? Can you design an input that requires $O(n)$ iterations? If so, describe the structure of this input. If not, argue why this is not possible.

   (c) Finally, can you design an input that takes fewer than n iterations? Why or why not?

   Aim for clarity and conciseness in your write up of this problem. You should have all the necessary tools to express your solutions. You do not need formal proofs or pseudocode, but you should be able to clearly articulate your ideas in plain English.

5. **(extra credit problem)** In lecture, we discussed why m is a lower bound for the Hiking problem. Show that $3m$ is a lower bound for the Hiking Problem.

Once you have completed your write-up *in this file*, double check this list:

- Please don't include your name in your submission. (Grading happens anonymously to minimize bias.)
- Make sure all your write-up details are in this `.tex file`. Make sure this `.tex` file is pushed to your github repository.
- Make sure this `.tex` file compiles on the CS lab computers. It is your responsibility to submit a file which compiles without errors.
- Don't submit a pdf! I will pull and compile the LaTeX. pushed all updates to the `.tex` file to your github repo (please don't submit a pdf)
- Once everything is done, fill out the post-homework survey.