CS41 Lab 7: Recurrence Relations

October 20, 2022

The lab problems this week focus on recurrence relations. Recurrence relations are important tools for modeling and analyzing the efficiency of algorithms. The purpose of this lab is to gain practice using the different techniques for solving recurrence relations.

The following equalities might be helpful:

- For any $a, b > 0$, $\log(a^b) = b \log a$.
- For any $a, b \geq 1$, $a^{\log b} = b^{\log a}$.
- For any $a, b \geq 1$, $\log(a \cdot b) = \log(a) + \log(b)$.
- For any $a, b \geq 1$, $\log(a/b) = \log(a) - log(b)$.
- For any $x \in \mathbb{R}$, $2^{\log x} = x$.

The following summation appears often in recurrence relations.

Fact 1. *Fix any constant $c > 0$. For all $m > 0$, we have*

$$\sum_{k=0}^{m-1} c^k = \frac{c^m - 1}{c - 1} \ .$$

Try to use both the Substitution Method and the Recursion Tree Method to solve these recurrence relations. You do not need to solve these exactly, only asymptotically. e.g. if a recurrence relation has an exact solution of $T(n) = 3n^2 - 19\sqrt{n}$, then an answer of $T(n) = O(n^2)$ suffices.

1. $S(n) = S(n-1) + 3n$,
   $S(1) = 3$
2. $M(n) = 3M(n/2) + 2n$,
   $M(1) = 1$
3. $W(n) = 3W(n/3) + n^2$,
   $W(1) = 1$
4. $H(n) = 4H(n/2) + 2n^2$,
   $H(4) = 5$
5. $T(n) = 3T(n/3) + 10\sqrt{n}$,
   $T(1) = 5$.