CS41 Homework 9

This homework is due at 4:59PM on Wednesday, November 23. Write your solution using LaTeX. Submit this homework in a file named `hw9.tex` using **github**.

This is a partnered homework. You should primarily be discussing problems with your homework partner. It's ok to discuss approaches at a high level with others. However, you should not reveal specific details of a solution, nor should you show your written solution to anyone else. The only exception to this rule is work you've done with a lab teammate *while in lab*. In this case, note (in your **homework submission poll**) who you've worked with and what parts were solved during lab.

1. Give a polynomial-time verifier for VERTEX-COVER.

2. MULTIPLE-INTERVAL-SCHEDULING In this problem, there is a machine that is available to run jobs over some period of time, say 9AM to 5PM.

   People submit jobs to run on the processor; the processor can only work on one job at any simgle point in time. However, in this problem, each job requires a **set of intervals** of time during which it needs to use the machine. Thus, for example, one job could require the processor from 10AM to 11AM and again from 2PM to 3PM. If you accept this job, it ties up your machine during these two hours, but you could still accept jobs that need any other time periods (including the hours from 11AM to 2PM).

   Now, you're given an integer k and a set of n jobs, each specified by a set of time intervals, and you want to answer the following question: is it possible to accept at least k of the jobs so that no two of the accepted jobs have any overlap in time?

   In this problem, you are to show evidence that MULTIPLE-INTERVAL-SCHEDULING is hard. To assist you, we've broken down this problem into smaller parts:

   (a) First, give a polynomial-time verifier for MULTIPLE-INTERVAL-SCHEDULING.

   (b) In the remaining two parts, you will reduce

   $$\text{INDEPENDENT-SET} \leq_P \text{MULTIPLE-INTERVAL-SCHEDULING} .$$

   Given input $(G = (V, E), k)$ for INDEPENDENT-SET, create a valid input for MULTIPLE-INTERVAL-SCHEDULING. First, divide the processor time window into m distinct and disjoint *intervals* $i_1, \ldots, i_m$. Associate each interval i_j with an edge e_j. Next, create a different job J_v for each vertex $v \in V$. What set of time intervals should you pick for job J_v?

   (c) Finally, run the MULTIPLE-INTERVAL-SCHEDULING algorithm on the input you create, and output YES iff the MULTIPLE-INTERVAL-SCHEDULING algorithm outputs YES. Argue that the answer to MULTIPLE-INTERVAL-SCHEDULING gives you a correct answer to INDEPENDENT-SET.

3. Three people enter a room and have a each have Garnet or Gray hat placed on their heads. They cannot see their own hats, but can see the other hats. The color of each hat is purely

random (i.e. w/probability 50% the hat is Garnet, w/probability 50% the hat is Gray). Each person needs to guess the color of their own hat by writing it on a piece of paper. They can also write "pass". They cannot communicate with each other in any way once the game starts but can discuss strategies before the game starts.

The players each win ONE MILLION DOLLARS if at least one player guesses their hat color correctly **and** no player guesses their hat color incorrectly. If all players pass, or if any player guesses incorrectly, players lose the game and win nothing.

Give a strategy that causes the players to win ONE MILLION DOLLARS with probabilty greater than 50%. Describe your strategy in words and rigorously analyze their winning probability.