CS41 Homework 10

This homework is due at 11:59PM on Wednesday, December 7. Write your solution using LaTeX. Submit this homework in a file named `hw10.tex` using **github**.

This is a partnered homework. You should primarily be discussing problems with your homework partner. It's ok to discuss approaches at a high level with others. However, you should not reveal specific details of a solution, nor should you show your written solution to anyone else. The only exception to this rule is work you've done with a lab teammate *while in lab*. In this case, note (in your **homework submission poll**) who you've worked with and what parts were solved during lab.

1. In the Four-Coloring problem, the input is a graph $G = (V, E)$, and you should output yes iff the vertices in G can be colored using at most four colors such that each edge $(u, v) \in E$ is *bichromatic*. Prove that Four-Coloring $\in$ NP-complete.

2. Path-Selection (K&T 8.9) Consider the following problem. You are managing a communication network, modeled by a directed graph $G = (V, E)$. There are c users who are interested in making use of this network. User i (for each $i = 1, \ldots, c$) issues a *request* to reserve a specific path P_i in G on which to transmit data.

   You are interested in accepting as many of these path requests as possible, subject to the following restriction: if you accept both P_i and P_j, then P_i and P_j cannot share any nodes.

   Thus, the Path-Selection problem asks: Given a directed graph $G = (V, E)$, set of requests $P_1, \ldots, P_c$—each of which is a path in G, and a number k, output yes iff it is possible to select at least k paths so that no two of the selected paths share any nodes.

   Prove that Path-Selection is NP-complete.

3. Intersection-Inference (K&T 8.16) Consider the problem of reasoning about the identity of a set from the size of ts intersections with other sets. You are given a finite set U of size n, and a collection $A_1, \ldots, A_m \subset U$ of subsets of U. You are also given integers $c_1, \ldots, c_m$. The question is: does there exists $X \subset U$ such that for each $i = 1, 2, \ldots, m$, the cardinality of $X \cap A_i$ equals c_i. We will call this an instance of the Intersection-Inference problem, with input $U, \{A_i\}, \{c_i\}$.

   Prove that Intersection-Inference is NP-complete, **Hint:** reduce from the following problem, which you may assume is NP-complete:

   Problem One-In-Three-Sat:

   Inputs: n variables $x_1, \ldots, x_n$ and m clauses $c_1, \ldots, c_m$ where each clauses is the or of three literals e.g., $c_i = (x_1 \vee \overline{x_2} \vee x_3)$.

   Output: yes iff there is a truth assignment to the variables such that for each clauses there is **exactly** one satisfied variable.

   Hint: Let U be the set of literals. You'll have to work to ensure that a variable and its negation cannot both end up in X.