Pumping Lemma:
Regular Languages

If A is a regular language,
then there is a pumping length p st
if $s \in A$ with $|s| \geq p$ then we can write $s = xyz$
so that

- $\forall i \geq 0 \; xy^iz \in A$
- $|y| > 0$
- $|xy| \leq p$

To prove A is *not* regular using the Pumping Lemma

1. Suppose A is regular
2. Call its pumping length p
3. Find string $s \in A$ with $|s| \geq p$
4. The pumping lemma says that for *some* split $s = xyz$ *all* the following conditions hold
   - $\forall i \geq 0 \; xy^iz \in A$
   - $|y| > 0$
   - $|xy| \leq p$
5. <u>Goal</u>: Find a string that violates the lemma

Pumping Lemma: Context Free Languages

If A is a context free language
then there is a pumping length p st
if $s \in A$ with $|s| \geq p$
then we can write $s = uvxyz$ so that

- $\forall i \geq 0 \; uv^ixy^iz \in A$
- $|vy| > 0$
- $|vxy| \leq p$

To prove A is *not* context free using the Pumping Lemma

1. Suppose A is context free
2. Call its pumping length p
3. Find string $s \in A$ with $|s| \geq p$
4. The pumping lemma says that for *some* split $s = uvxyz$ all the following conditions hold
   - $\forall i \geq 0\ uv^ixy^iz \in A$
   - $|vy| > 0$
   - $|vxy| \leq p$
5. Goal: Find a string that violates the lemma

To prove $\{a^n b^n c^n \mid n \geq 0\}$ is *not* context free using the Pumping Lemma

1. Suppose $\{a^n b^n c^n \mid n \geq 0\}$ is context free
2. Call its pumping length p
3. Find string $s \in A$ with $|s| \geq p$. Let $s = a^p b^p c^p$
4. The pumping lemma says that for *some* split $s = uvxyz$ all the following conditions hold
   - $\forall i \geq 0 \; uv^i xy^i z \in A$
   - $|vy| > 0$
   - $|vxy| \leq p$

To prove $\{a^n b^n c^n \mid n \geq 0\}$ is *not* context free using the Pumping Lemma

- Suppose $\{a^n b^n c^n \mid n \geq 0\}$ is context free.
- Let $s = a^p b^p c^p$
- The pumping lemma says that for *some* split $s = uvxyz$ all the following conditions hold
  - $|vy| > 0$
  - $uvvxyyz \in A$

<u>Case 1:</u> both v and y contain at most one type of symbol

<u>Case 2:</u> either v or y contain more than one type of symbol

To prove $\{a^i b^j c^k \mid 0 \le i \le j \le k\}$ is *not* context free using the Pumping Lemma

1. Suppose $\{a^i b^j c^k \mid 0 \le i \le j \le k\}$ is context free
2. Call its pumping length p
3. Find string $s \in A$ with $|s| \ge p$. Let $s = a^p b^p c^p$
4. The pumping lemma says that for *some* split $s = uvxyz$ all the following conditions hold
   - $\forall i \ge 0\ uv^i xy^i z \in A$
   - $|vy| > 0$
   - $|vxy| \le p$

To prove $\{a^i b^j c^k \mid 0 \leq i \leq j \leq k\}$ is *not* context free using the Pumping Lemma

- Suppose $\{a^i b^j c^k \mid 0 \leq i \leq j \leq k\}$ is context free.
- Let $s = a^p b^p c^p$
- The pumping lemma says that for *some* split $s = uvxyz$ all the following conditions hold
    - $uvvxyyz \in A$
    - $|vy| > 0$

Case 1: both v and y contain at most one type of symbol

Case 2: either v or y contain more than one type of symbol

To prove $\{a^i b^j c^k \mid 0 \le i \le j \le k\}$ is *not* context free using the Pumping Lemma

- Let $s = a^p b^p c^p$
- *some* split $s = uvxyz$
- $|vy| > 0$

<u>Case 1:</u> both v and y contain at most one type of symbol

(i) a's not included $\Rightarrow uxz \notin A$

(ii) b's not included

  (i) a's included $\Rightarrow uvvxyyz \notin A$

  (ii) c's included $\Rightarrow uxz \notin A$

(iii) c's not included

  (i) a's included $\Rightarrow uvvxyyz \notin A$

  (ii) b's included $\Rightarrow uvvxyyz \notin A$